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Application of a single-equation SARIMA model for short-term conditional forecast (projection) of CPI price dynamics in Poland

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Abstract

The aim of the article is to construct an optimal SARIMA model for short-term conditional forecasting (projection) of price dynamics expressed by the Consumer Price Index (CPI), as understood within the extended Box-Jenkins procedure. The construction of such a forecast aims to influence, through the expectations channel, the institutional trust of society in monetary authorities and assess the effectiveness of achieving the monetary policy goal within the framework of the democratic responsibility of the decision-making body of the National Bank of Poland – the Monetary Policy Council. The selection of the optimal SARIMA model was carried out using an iterative method within the Box-Jenkins procedure, with the goal of reducing the systematic bias of estimators – coherence with empirical data. The analysis was conducted on compiled secondary data of the monthly Consumer Price Index for goods and services from Statistics Poland (formerly: Central Statistical Office) for the years 2010-2023 (on a monthly basis). Results show that the short-term forecast demonstrated accuracy within a specified confidence interval. The application of the SARIMA model serves as a useful methodological tool for constructing elaborate DSGE models (for example, the NECMOD model) using procedures such as SEM (System for Averaging Models) from Norges Bank.

Keywords

- SARIMA
- forecasting
- CPI
- inflation targeting strategy
- monetary policy

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Introduction

The demand for the application of analytical tools through a quantitative description of price formation processes in the economy of a given country was indicated in the 20th century by the creator of the term "econometrics" – Paweł Ciompa (Gruszczyński, 2022). He was the author of a series of publications, including "Drożyzna w Galicyi i nędza urzędnicza", in which he identified the inflationary imbalance caused by the accelerated pace of relative price growth, exceeding the wage adjustments of workers in branches producing lower-order goods in the then-eastern part of the Austrian partition. The colloquial term "inflation" (of prices), as the increasing rate of change in absolute prices present in contemporary political economy discourse, is nothing more than the loss of purchasing power of a unit of wages at the current level of nominal production.

The genesis of inflationary imbalance in Poland, indicated by the Consumer Price Index (CPI) exceeding the upper limit of permissible deviation from the reference value, occurred in April 2021 when the relative value of CPI year-on-year reached 4.3%. The first instance of not achieving the overarching goal of monetary policy above the established nominal anchor with a symmetrical deviation range, since September 2012, took place in January 2020, with the relative value of the CPI year-on-year at 4.3%. However, the actual failure to meet the monetary policy target occurred during the period of deflationary trends from November 2018 to February 2019. The period of inflationary imbalance from January to March 2020 was mainly a result of government triangular intervention through the increase in the minimum wage to PLN 2,600 by the Regulation of the Council of Ministers of 2019. According to reports from Statistics Poland (formerly: Central Statistical Office) (GUS, 2021), this period also witnessed price adjustments in the sectors of food and non-alcoholic beverages (a decrease of –24.89% compared to January of the previous year) and housing and household operation (similarly – 19.17%). Although the Annual Report of the National Bank of Poland for 2020 (NBP, 2021) unequivocally attributed the increasing pace of CPI (year-on-year) to the negative supply shock, for which monetary policy had no direct reaction channel, there was also a clear upward trend in core inflation in 2020. The economic contraction period due to the COVID-19 pandemic led to a significant drop in energy and commodity prices, along with the appreciation of the nominal exchange rate of the Polish złoty against major trading partners from March 2020. In April 2021, the estimate from Statistics Poland showed a deviation from the reference value of the overarching monetary policy target – CPI inflation (year-on-year) at 4.3%, following a year of price stability (direct achievement of the inflation target with a symmetrical deviation range). The stimulating factor for inflationary adjustment was the upward trend in commodity prices, particularly oil, as emphasised in the NBP's annual report. According to the report by Statistics Poland, the trade balance in 2021 was negative, amounting to approximately PLN 7 billion (GUS, 2022). This trade balance passivation also affected the worsening terms of trade and the average monthly real exchange rate of USD/PLN by approximately 8.02% compared to December of the previous year. In 2022, due to the armed aggression of the Russian Federation against Ukraine, commodity markets (especially futures contracts) experienced further price increases for oil (Brent and WTI) and gas, further amplifying the price adjustments resulting from the deepening passivation of the trade balance (in 2022, a negative trade balance was recorded at PLN 92.5 billion). Currently, the period of inflationary imbalance in Poland is driven by the accommodating fiscal policy, empirically verified by the estimates of the National Bank of Poland regarding core inflation, which indicated a continuously growing price dynamics from June 2021 (3.5% compared to the previous year) to March 2023 (12.3%). Empirical data clearly point to the significant impact of bottlenecks as a leading variable stimulating the pace of price changes in the Polish economy. It is essential to note that since December 2021, the National Bank of Poland has initiated a deflationary policy by reducing excess liquidity from the banking sector through the application of liquidity-absorbing fine-tuning open market operations. In addition to direct quantitative tightening through absorbing fine-tuning operations in December 2021, aimed at unifying the POLONIA rate to the level of the NBP reference rate, there was a series of increases in the basic interest rates (NBP, 2023) from October 2021 (keeping the deposit rate of basic open market operations unchanged at 0% until November 2021, when the volatility range of market interest rates was raised by 75 basis points) to September 2023. However, the cycle of raising the basic interest rates proved to be ineffective in the deflationary policy due to the disparity in the conducted "policy mix" in Poland. The income effect surplus, resulting from fiscal impulses in Poland and triangular intervention in the form of minimum wage regulations, over the NBP's "austerity" policy, although effective countercyclically (GUS, 2023a) (with a year-on-year real GDP growth of 6.9% in 2021 and 5.1% in 2022), was a leading stimulant of inflationary imbalance. The above observation of discretionary "policy mix" in Poland allows for the identification of the impact of fiscal impulses on real and nominal parameters with a shorter external lag compared to monetary policy, positively verifying the principle of the primacy of fiscal policy.

As a result of the national referendum on Poland's accession to the European Union on June 7–8, 2003, and the subsequent signing of the accession treaty (Athens Treaty) on May 1, 2004, Poland committed itself to adopting the principles of the Economic and Monetary Union (EMU), which is a formal structure within the concept of integration of member states under the Maastricht Treaty (Treaty on European Union signed on February 7, 1992). The formal commitment arising from the status of a member state of the European Union involves joining the Economic and Monetary Union, fulfilling one of the nominal convergence criteria established under Article 140(1) of the Treaty on the Functioning of the European Union. A member state maintaining derogation status in the process of currency integration within the EMU structure is subject to a report prepared by the European Commission and the European Central Bank every two years or upon the request of a member state under derogation status, assessing the progress of the nominal convergence procedure for that specific country. According to the Convergence Report for 2022 (European Commission, 2022), the reference value for the most stable dynamics of prices measured by the Harmonised Index of Consumer Prices (HICP), according to Eurostat methodology, was set at 4.9% (with an allowable symmetrical deviation range of 1.5 percentage points according to Protocol No. 13 on Convergence Criteria). The indicator of the pace of absolute price changes HICP (12-month average HICP) for the basket of goods in Poland reached 7% in April 2022, according to Eurostat estimates, which does not meet the nominal convergence criterion set out in the Treaty on the Functioning of the European Union. The baseline dynamics of absolute prices, excluding the impact of price adjustments due to bottlenecks, showed a value of 5.3%. Among the main factors stimulating the pace of price changes were the service sector (change – 7.6%) and energy (change – 18.2%). It is important to emphasise that in April of the following year, HICP for Poland reached the value of 15.2%.

The aim of the work is to develop an optimal adaptive SARIMA model using the extended Box-Jenkins method (Box & Jenkins, 1983) with a seasonal component, an explanatory model's indicator (Kashpruk, 2021), and an assessment of the ex post prediction accuracy (Romanuke, 2022). The extended procedure is intended to apply the optimal SARIMA model for a reliable analysis and accurate conditional forecasting (projection) of the Consumer Price Index (CPI) year-on-year dynamics for the Polish economy.

The extended procedure for constructing and estimating the SARIMA model was conducted using data from Statistics Poland (GUS, 2023b) on the change in the year-on-year rate of absolute price changes – "inflation" CPI (y/y) from 2010 to 2023 on a monthly basis. The obtained secondary data show a sample of 223 observations for the examined time series. Estimates for the conducted verification tests and generated visualisations of the estimation were carried out in the Rstudio environment.

1. Literature review

The Inflation Targeting strategy is a multiparametric approach in which central banks use a reaction function through available monetary policy instruments (both standard and unconventional) to implement a monetary strategy with discretionary decisions regarding the scale and time frame (which determines the degree of flexibility in implementing the DIT strategy) of monetary policy target achievement. The multiparametric strategy is informationally inclusive, taking into account a broader set of parameters influencing the change in the price level. This strategy involves the analytical identification by monetary authorities and their expert bodies, such as the decision-making body (in Poland, the Monetary Policy Council), and analytical departments, of variables that determine deviations from the established equilibrium point of the nominal anchor. Although the implementation of the Direct Inflation Targeting strategy is multiparametric, the National Bank of Poland (the declaration of the monetary strategy was made in October 1997. Formal introduction of the inflation target took place in February 1998 by the newly established decision-making body – the Monetary Policy Council), concluding the period of systemic and banking system transformation from a monobank to relative independence of the banking sector in a universal model from the central bank, pursued the classical triad of monetary policy goals, with the final goal being the reference value of CPI (y/y) inflation dynamics (Pietryka, 2008). Since 2004, at the request of the Monetary Policy Council composition, the National Bank of Poland has been implementing the Direct Inflation Target with a reference value of 2.5% and a symmetric deviation range of 1 percentage point, gradually reducing the reference value from 8% in 1999. A significant change in the technical approach to achieving the target criterion in 2004 was the establishment of the reference value of price dynamics as a continuous value, rather than an annual one. Since then, the National Bank of Poland has adopted the monthly CPI relative to the previous year (y/y) as the nominal anchor, whereas previously, it was the annual average value adopted as the value in December relative to the previous year. The reduction of the reference value of the nominal anchor by the NBP decision-making body aimed to maintain coherence with the conducted disinflationary policy after the period of systemic transformation. The Monetary Policy Council, identifying the process of spillover effects as the diffusion of institutional and economic integration with foreign countries, not only expanded the framework of discretionary monetary policy but also recognised the impact of supply shocks on price dynamics. Accordingly, it decided to raise the reference value of the final target again in 2001. However, in 2000, anticipating the impact of bottlenecks on price adjustments and recognising the accommodative fiscal policy, it decided to lower the reference value. The National Bank of Poland, committing to achieving the established target criterion, is subject to the assessment of society through institutional trust and democratic responsibility. This responsibility determines the degree of societal trust in the central bank through the evaluation of the achievement of the overarching goal of monetary policy. Furthermore, in the case of the National Bank of Poland, democratic responsibility lies with the decision-making body – the Monetary Policy Council and the President of the National Bank of Poland, indicating a collegial model of central banking in Poland. In the case of the National Bank of Poland, the implementation of the DIT strategy over time is flexible, as monetary authorities declare the reaction function over the medium-term horizon (Ancyparowicz, 2017).

The strategy of direct inflation targeting as a goal criterion of monetary policy, along with the reference value determining the target function, was first introduced by the Reserve Bank of New Zealand in 1990 (Gatnar, 2018) and the Central Bank of Chile (Mishkin & Schmidt-Hebbel, 2002), although it is officially accepted that Chile adopted the DIT strategy in 1999 (Banco Central de Chile, 2007). In subsequent years, central banks adopted DIT strategies with a symmetric deviation range or a deviation range, a point-target DIT strategy (a point determined by a constant reference value of inflation dynamics), as in the case of the Bank of England in 1992 (2%), or an asymmetric deviation range from the reference value, as in the case of the European Central Bank (2% and below). As indicated in Figure 1, selected central banks only introduced the Direct Inflation Targeting

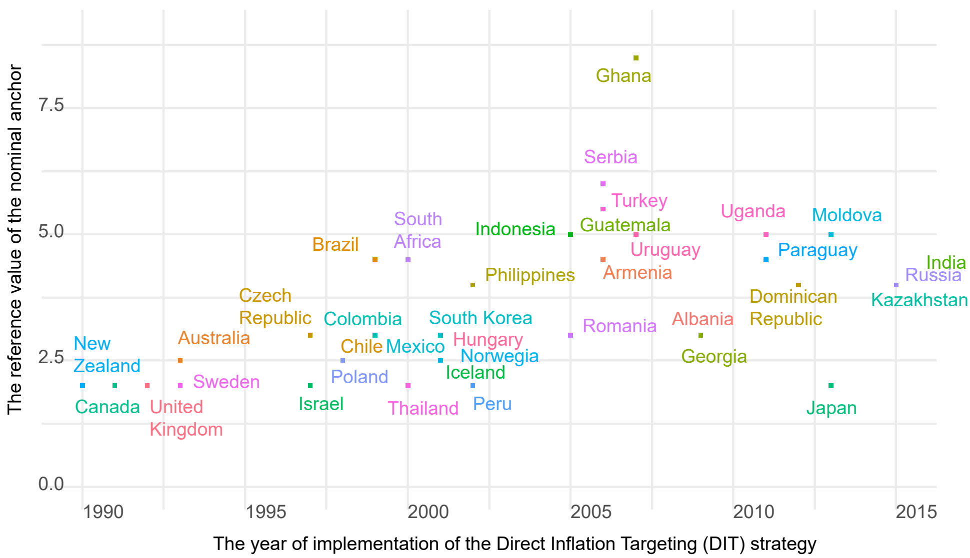

Figure 1. Decisions to adopt the inflation targeting strategy by selected central banks in the years 1990–2015

Source: own compilation based on data from (Roger, 2010; Hammond, 2011; Ciżkowicz-Pękała et al., 2019).

strategy as a goal criterion and the target function of monetary policy in 2015, including Russia, Kazakhstan and India.

Central banks, using analytical estimates and projection construction tools, influence the credibility and effectiveness of their monetary policy strategy through the expectations channel. The implementation of a monetary strategy, conditioned on achieving the stabilisation requirement of a chosen target value (conditioned by the goal criterion) as the nominal anchor of its reference level, belongs to the set of components (accountability) of the central bank's democratic responsibility (Matysek-Jędrych, 2014). This democratic responsibility does not only take into account qualitative aspects of the institutional image and societal trust in the central bank but also undergoes societal assessment through the execution of a monetary strategy conditioned by a specific measurable value – the policy target criterion. Communication with society in a collegial model of monetary policy occurs through the procedure of voting on the proposed value established as the target criterion of the monetary strategy. This is a communication channel conditioned by the transparency component. The qualitative aspect of societal trust, credibility, is a component that conditions the feedback loop with the independence aspect of the monetary policy decision-making body and determines (reduces or amplifies) the alternative cost of monetary authorities' reaction function to the chosen target criterion on other parameters. The alternative cost of the monetary strategy relative to the chosen nominal anchor (Rule of Thumb) was formulated by the Federal Reserve (Federal Reserve Board, 1996) to describe the difference in identifying responses to monetary impulses (quantitative tightening) within the framework of conducted disinflationary policies between the vector autoregression VAR model and the full macroeconometric DSGE model of the Federal Reserve – FRB/US. The construction of forecasts for selected macroeconomic parameters can be divided into two types: conditional forecast (projection) and unconditional forecast. Both forecast variants differ in the methodological approach regarding the identification condition in the monetary authorities' reaction function model (unconditional forecast). For example, identifying the change in the current (monetary) interest rate endogenously shaped by expectations of economic agents (liquidity preference) is an approach of the conditional forecast variant, which does not assume values such as the relative change in the CPI price growth rate in the analysed period (or periods) subject to projection. Sławiński (2011) defines the conditional forecast variant of the direct inflation targeting strategy as "a forecast assuming no changes in interest rates in the forecast horizon". Of course, "change in interest rates" is understood as a change in the central bank's reference rates. As emphasised by Misztal (2023), the most commonly used and simplest method of measuring credibility is the difference between the model's expected value (or inflation expectations) and the actual CPI inflation rate.

The measure of the absolute price change rate based on the basket of goods and services within the analysed "inflation basket" is not uniform across public institutions responsible for collecting and processing data in various countries for the purpose of societal statistical analysis and public institutions. Inflation rate indices take into account not only the change in price expressed in monetary units, which is a liquid medium retaining the function of a universal means of payment (whose condition of elementary primary exchange value is institutional trust) and a record-keeping function (the ability to scale and express in monetary units the objective utility value of a good or service – the supply price), but also changes in the supply of goods and services offered in economic transactions. The Consumer Price Index (CPI) estimated in Poland for the inflation basket by Statistics Poland follows a methodology similar to Eurostat (2018) for the Harmonised Index of Consumer Prices (HICP), based on the formula of the aggregated Laspeyres price index. The formula of the aggregated Laspeyres price index is based on the change in the price level in the current period, *t*, compared to the base period, *t*–1, which methodologically differs from the aggregated Fisher price index used by the US Bureau of Labour Statistics (BLS), which synthesises the methodologies of both Laspeyres and Paasche – as the root of the products of both indices. The generalised Fisher index is adjusted by a parameter that is the exponent, α, for the chosen index. Where the parameter α = 0.5 is the classical Fisher index (Białek, 2014). The Laspeyres index is an indicator that measures the change in the price level, as the periodic difference concerns the component of price change. In the case of the Paasche index, the measure is of the change in the value of the consumer basket of goods and services. Therefore, the Laspeyres index is resistant to changes in the volume of the supply of goods and services in the market. The Laspeyres index takes the following form:

$$I_L = \frac{\sum_{i=1}^{N} p_i^t q_i^s}{\sum_{i=1}^{N} p_i^s q_i^s}$$

where:

$p_i^t = \left\{ p_1^t, p_2^t, \ldots, p_N^t \right\}$ – price vector for the current period t,

$q_i^t = \left\{ q_1^t, q_2^t, \ldots, q_N^t \right\}$ – quantity vector for the current period t,

$p_i^s = \left\{ p_1^s, p_2^s, \ldots, p_N^s \right\}$ – price vector for the current period s,

$q_i^s = \left\{ q_1^s, q_2^s, \ldots, q_N^s \right\}$ – quantity vector for the current period s.

The measure of the CPI indicator in Poland is adjusted by a weighting system (see Table 1) (Hałka & Leszczyńska, 2011), which assigns certain weights to specific goods and services. These weights are estimated based on the shares of individual goods and services in overall consumer expenditures in the previous period. However, the CPI index does not remain an ideal (unbiased) indicator that would account for the change in relative prices resulting from the diffusion of their level through the impact of specific impulses (including the transmission of monetary flows) within the first-round effect. The "diffusion of price levels" (Keynes, 1930) distorts the weighting system for calculating the marginal utility of specific goods and services, consequently reducing the resistance of the CPI index to price changes in branches of production of heterogeneous goods (due to consumer preference effects) that include a percentage markup on unit costs, as indicated by the Ramsey rule (Raa, 2009).

Table 1. The weighting system used in estimating the CPI index by Statistics Poland in Poland

Respective	2019	2020	2021	2022	2023
Food and non-alcoholic beverages	24.89	25.24	27.77	26.59	27.01
Alcoholic beverages and tobacco products	6.37	6.25	6.91	6.32	5.75
Clothing and footwear	4.94	4.94	4.21	4.47	4.27
Housing and utility costs	19.17	18.44	19.14	19.33	19.63
Household furnishings and appliances	5.70	5.80	5.83	5.71	5.29
Healthcare	5.12	5.29	5.39	5.69	5.71
Transportation	10.34	9.89	8.88	9.54	9.92
Communication	4.18	4.54	5.00	4.90	4.48
Recreation and culture	6.44	6.62	5.78	6.07	6.14
Education	1.07	1.15	1.02	1.16	1.21
Restaurants and hotels	6.20	6.12	4.56	4.77	5.11
Other goods and services	5.58	5.72	5.51	5.45	5.48

Source: own elaboration based on GUS data.

White (1999) points out the main sources of measurement bias in the relative value change rate of the CPI index based on the Laspeyres index:

1. Systematic measurement bias: Measurement bias arising from the adopted measurement methodology pertains to the data aggregation stage and the procedure of averaging the prices of goods and services at various points of sale (especially at the lowest level of aggregation).
2. Measurement bias resulting from the substitution effect in the production branches of homogeneous goods.
3. Measurement bias resulting from changes in the qualitative characteristics of individual goods and services.

4. Measurement bias resulting from the introduction of new goods and services to the market.
5. Measurement bias from the substitution of markets: Measurement bias arising from new sources of acquiring goods and services and discrepancies in the prices of individual goods.

The measurement bias of the CPI index corrected for the weighting system of the Laspeyres index for individual goods is the result of the aforementioned delays in the time horizon relative to the imposed weights as a surrogate for the marginal utility of the consumer concerning individual goods and services in the inflation basket. Therefore, the Laspeyres index aims to approximate the real utility of individual goods and services expressed by the Cost of Living Index (COLI).

In the analysis dedicated to the CPI index bias for Poland in the scientific work of Hałka and Leszczyńska (2011), it is essential to identify the underestimation of the CPI index using the Laspeyres index method, mainly due to the lack of adjustment of the index for the substitution effect, the operation of the "plutocratic gap" and, as the authors point out, the impact of demand rigidity in branches producing heterogeneous goods (which empirically verifies the susceptibility of the index to the degree of monopolisation in specific branches of production of autonomous consumption goods).

The single-equation linear ARIMA (AutoRegressive Integrated Moving Average) model, which directly originates from the work "Time Series Analysis: Forecasting and Control" by G. E. P. Box and G. M. Jenkins, is, according to the typology of macroeconometric model classification (Hara et al., 2009), a time series model. However, the specificity of the model distinguishes its structure from other models, as the construction stage of the model is based on the empirical identification and optimisation of the model (as a linear combination of parameters of autoregressive and moving average processes under specified assumptions regarding the integration process) based on a time series of a particular macroeconomic variable. In contrast to multivariate models, whose construction is based on the selection of estimated parameter matrices, single-equation ARIMA models do not require empirical reduction of identification bias in the model or diagnosis of causality between selected structural parameters of the models. ARIMA models, focusing on the empirical identification of the time series of a selected variable, serve as analytical tools for conditional forecasting, where the processes shaping events not captured by the model are endogenous within the system and integrated with the studied variable, which itself is integrated with the information flowing from the system. Although ARIMA models do not identify the direct impact of changes in exogenous factors (such as structural shocks caused by "policy mix" impulses or supply shocks) and represent an example of an analytical tool for conditional forecasting, as emphasised by Yusifov (2014), ARIMA models, whose construction is based on the stochastic properties of the studied variable, are resistant to Lucas Critique. The extension of ARIMA models to nonlinear identification of the stochastic process of conditional variance through the synthesis of a GARCH-class model – ARIMA-GARCH (Zhou et al., 2006) allows for more accurate forecasting of a selected parameter and risk mitigation, addressing the systematic error burden of linear combination models in the ARIMA class. An analytical tool, especially recommended by Eurostat (2009), for comprehensive estimation of an optimal SARIMA model (seasonal adjustment) is TRAMO-SEATS (Time Series Regression with ARIMA noise, Missing values and Outliers & Signal Extraction in ARIMA Time Series). The TRAMO-SEATS method synthesises two methodologies for the optimal estimation procedure of the SARIMA model:

- estimation of the ARIMA model with the identification of outliers, missing data correction (observations) and leverage value correction (e.g. to adjust the stochastic process for reducing impulses resulting from calendar effects) – TRAMO procedure (Time series Regression with ARIMA noise, Missing values and Outliers),
- decomposition of the linearised time series[2] through the identification of the harmonic structure (spectral analysis of the spectral density function of the time series) of components (orthogonal to each other) of the time series; systematic component (trend), seasonal component and irregular component.

Statistics Poland currently predominantly utilises the TRAMO-SEATS procedure within the JDEMETRA+ package for seasonal adjustment of time series in quarterly and monthly sequences, including average employment, average monthly earnings and the Labour Force Survey (quarterly).

Time series models, including ARIMA models, though they serve as reliable methodological tools for identifying quantitative relationships and understanding the impact of time on the development of selected parameters with reduced systematic error (through optimisation procedures and diagnostic indicators), are sensitive to the replicability of studies and the forecasting horizon. This sensitivity is mainly due to the influence of exogenous factors such as “policy mix” or “black swan” events, where extrapolation is sensitive to shocks. This constitutes a dimension to be considered in the macroeconomic time series modelling within the framework of Lucas Critique. The analytical foundation of the decision-making body of Norges Bank utilises a system of averaging SAM models through the optimisation procedure of the DSGE model – NEMO (Norwegian Economy Model). This

[2] As indicated by such authors as Grudkowska and Paśnicka (2007), a time series, before undergoing the SEATS procedure, undergoes linearisation. In this process, the time series in its original form is logarithmised and then decomposed into the sum of a seasonal component and a non-seasonal component.

involves constructing the probability density distribution of estimated outcomes of short-term conditional forecast models. There are 172 models for short-term projections of "core inflation" (CPI-ATE) and 221 for the conditional GDP forecast (Gerdrup & Nicolaisen, 2011). In this way, before constructing the DSGE – NEMO model, which maintains strong coherence with the New Neoclassical Synthesis theory[3], Norges Bank builds short-term forecasts that are subject to the SAM model averaging system. This process involves identifying exogenous assumptions and a heuristic technique that makes assessment and verification through the expertise of Norges Bank. The NEMO model of Norges Bank is a macroeconometric model using empirical data on a quarterly basis. Short-term forecasts (projections) based on such models as vector autoregression (161 models), indicator-based (5 models) and factor-based (5 models) provide forecasts for a horizon of up to 5 observations (5 quarters). The NEMO model constructs a conditional forecast for 8 to 12 quarters, considering supply blocks (Kravik & Mimir, 2019) and exogenous factors, including government (impact of fiscal impulses and triangular interventions), energy market (oil), foreign trade block, as well as the banking sector block.

Alan Greenspan (2008), the former chairman of the Federal Reserve from 1987 to 2006, penned a highly insightful article in the Financial Times titled "We will never have a perfect model of risk". In this article, he pointed out the main cause of the systematic errors in macroeconometric models – expectations. Although DSGE models introduce microeconomic foundations that identify the roles of expectations[4] and aim to maximise the marginal utility function of household consumption, they derive these foundations from neoclassical theory, which does not account for the heterogeneity of expectations[5]. Furthermore, these models (DSGE, similar to Cowles Commission structural models before Sims' critique) are unable

[3] Described by Norges Bank as a "New Keynesian" model (Tura, 2011), it incorporates conflicting assumptions, including rational expectations, wage and price stickiness, Walras's law and information asymmetry.

[4] These are the expectations of rational agents, which do not account for adaptive expectations, information asymmetry and animal spirits.

[5] The attention to the heterogeneity of expectations, which constitutes the foundation for the volitional calculation of individuals (Mises, 2011), is emphasised by Kenneth Arrow: "One of the things that microeconomics teaches you is that individuals are not alike. There is heterogeneity, and probably the most important heterogeneity is heterogeneity of expectations" (Hommes, 2005, p. 2). The production process resulting from a sequence of subjective expectations determining time preferences determines the demand for the extension of stages in the production structure ("cumulative process" in this case, according to Wicksell, will persist until the marginal efficiency of capital equals the monetary interest rate), within the Ricardian effect (Ruys, 2017). This is done not only to expand the real stock of intermediate goods but also to deepen capital. The goal of deepening capital is based on the demand for a lasting stock of intermediate goods. Moreover, this reflects the endogenous nature of introducing innovations into the production process and emphasises the qualitative character of identifying the heterogeneity of stages in the production structure, contrary to the homogeneous structure of capital in the classical-neoclassical school.

to identify crises caused by exogenous factors, the so-called "black swans" (including wars, political trends, pandemics and psychological aspects of mass behaviour). The experience of Norges Bank brings a significant tool to macroeconometric modelling, involving the implementation of an inductive-deductive approach of expert[6] background and adjusting the long-term forecasting model NEMO with the probability density of the conditional short-term forecasts from various time series models. In summary, SARIMA models can serve as a useful tool for short-term conditional forecasting at the initial stage of the NECMOD model construction procedure, which requires re-estimation not only in terms of theoretical coherence but also empirical coherence.

2. Research method

The elementary condition for the optimal selection of parameters in the autoregressive and moving average processes is the execution of tests that identify the optimal model for a time series with a stationary process. Stochastic processes can be categorised into strictly stationary (narrow-sense stationarity) or weakly stationary (broad-sense or covariance stationarity) processes. Weakly stationary processes are challenging to identify, as they require the joint probability density of the stochastic process $\{Y_t\}$ for any observation $\{t_1, t_2, \ldots, t_n\}$ to be equal for the realised processes; $\{Y_{t1}, Y_{t2}, \ldots, Y_{tn}\}$ and those shifted in time by $k\{Y_{t1+k}, Y_{t2+k}, \ldots, Y_{tn+k}\}$. For the inferential reasoning of tests identifying the parameters of the ARIMA model, the realisation of a weakly stationary process is often sufficient, ensuring that the conditions for the stability of central tendency statistics – mean and variance dispersion – are met:

$$\textit{Mean condition: } \mu_t = \mu$$
$$\textit{Variance condition: } \sigma_t^2 = \sigma^2$$
$$\textit{Covariance condition: } \gamma_{k,t} = \gamma_k$$

where:

$$E\left[(Y_t - \mu)(Y_{t-k} - \mu)\right] = \gamma_k,\ k = 1, 2, 3, \ldots$$

[6] Norges Bank's use of a short-term conditional forecasting model with vector autoregression based on subjective probability of Bayesian reasoning and a heuristic approach to prediction evaluation fulfils Keynes's proposition in *A treatise on probability* (Keynes, 1921) to imply specific a priori assumptions in the form of qualitative analysis instead of approximating reality with a certain error calibration of the econometric model (which was particularly the cause of Keynes's methodological dispute with Tinbergen).

The hypothesis regarding the stationarity of a time series is subject to verification after estimating tests that examine the presence of a unit root or investigate the stationarity of the time series. Accordingly, stationary tests can be divided into two groups (Miłek, 2017). The first group, examining the presence of a unit root, includes tests such as the DF (Dickey-Fuller test), ADF (Augmented Dickey-Fuller test), AD (Adki-Dickey test), with their permutation versions, as well as the Perron-Phillips test (Phillips & Perron, 1988). The second group, examining the stationarity of a time series, includes tests such as KPSS (Kwiatkowski-Phillips-Schmidt-Shin test) and versions designed to identify a stationary process for the seasonal component; the CH test (Canova-Hansen), HEGY (Hylleberg et al., 1990), ADFSI (ADF Seasonal Integration test), and DHF (Dickey et al., 1984). In the case of a zero-degree integration process ($Y_t \sim I(0)$), meaning the integration parameter $d = 0$, the time series is stationary. Conversely, if the hypothesis of non-stationarity of the time series is accepted, then the time series process is integrated to the first degree ($Y_t \sim I(1)$, corresponding to $d = 1$). In this case, first-order differencing is performed to transform the time series into a stationary form: $\Delta Y_t = Y_t - Y_{t-1}$. However, if the test statistic value still indicates the presence of a unit root or non-stationarity in the first--order differenced time series, second-order differencing should be conducted, involving differencing of differences: $\Delta^2 Y_t = \Delta Y_t - \Delta Y_{t-1}$. For a thorough analysis of the time series integration process, tests such as ADF (H_0: the process is integrated to the first degree) and KPSS (H_0: the time series is stationary) will be conducted to identify the parameter d. The selection of tests from both groups is based on the sensitivity of both tests, where the ADF test is sensitive to the persistence (autocorrelation process) of the time series, the occurrence of structural innovations or the presence of outliers. Meanwhile, the KPSS test is sensitive to replicability and the sample size, which may pose challenges in constructing an iterative procedure for the optimal selection of parameters for adaptive ARIMA class models.

In the classical Box-Jenkins procedure, the identification stage of constructing a single-equation adaptive ARIMA model involves selecting the autoregressive process parameter (p) and the moving average process parameter (q) by identifying them using the partial autocorrelation function (PACF, which allows for the identification of partial autocorrelation effects) and the autocorrelation function (ACF, which allows for the identification of disturbance effects) of the time series observations. The Partial Autocorrelation Function estimates the monotonic relationship between the observation at the current time period t, and the observation at time period $t - k$, with a reduction in the interaction of observations between the two periods, in such a way that:

$$PACF_{(y_t, y_{t-k})} = \frac{Cov\left(y_t, y_{t-k} \mid y_{t-1}, y_{t-2}, \ldots, y_{t-k+1}\right)}{\sigma_{y_t|y_{t-1}, y_{t-2}, \ldots, y_{t-k+1}} \sigma_{y_{t-k}|y_{t-1}, y_{t-2}, \ldots, y_{t-k+1}}}$$

Estimating the autoregressive process parameter, in the case of statistical significance for the partial autocorrelation of observations in the time series at a given lag, should be considered as the preliminary selection of the optimal level of the parameter p (when the time series information assumes the character of a stationary process, and in reality, most commonly it takes the form of covariance stationary[7]):

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + \varepsilon_t$$

The autocorrelation function helps identify the moving average process and select the parameter q as a sequence of disturbances (errors) on the information series. The moving average process of the MA model is a weighted average realisation of white noise over time. Accordingly, autocorrelation is determined as the ratio of covariance Y_k to variance Y_0, with k representing the lag:

$$ACF = r_{t,s} = r_k = \frac{\sum_{t=k+1}^{k} (Y_t - \overline{Y})(Y_{t-k} - \overline{Y})}{\sum_{t=1}^{k} (Y_t - \overline{Y})^2} = \frac{cov(Y_t, Y_{t-k})}{var(Y_t)} = \frac{\gamma_k}{\gamma_0}$$

where:

$\{Y_t : t = 0, \pm1, \pm2, \pm3 \ldots\}$, $t \in R$ – univariate time series representing the realisation of a stochastic process,

$n = |t - s|$, $t, s = 0, \pm1, \pm2, \pm3 \ldots$ – lag order.

The form of the MA model for the determined lag parameter of the moving average process is given by:

$$Y_t = \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t$$

The maximum likelihood method applied in the estimation of the model in this study is a statistical technique that involves minimising the variance of the error term and maximising the logarithm of the likelihood function. Accordingly, the logarithm of the likelihood function is estimated as follows:

$$\ln\mathcal{L}(\phi, \theta, \sigma_\varepsilon^2) = -\frac{N}{2}\ln 2\pi\sigma_\varepsilon^2 - \frac{\sum_{t=-M}^{n} \left[E\left(\varepsilon_t | \phi, \theta, Y\right) \right]^2}{2\sigma_\varepsilon^2}$$

[7] More commonly, a time series is transformed into a process of first differences, becoming a differenced-stationary process (Hamulczuk et al., 2011).

where:

σ_ε^2 – variance of the residual component for specified parameters of the autoregressive and moving average processes,

M – constant satisfying the condition that the absolute difference between the expected values of Y_t and Y_{t-1} is smaller than the random component.

The classical Box-Jenkins procedure for constructing ARIMA models, consisting of the stages of identification, estimation and diagnosis, does not optimise the model by reducing the bias of the estimators. Therefore, the extended procedure, incorporating explanatory indicators, allows for the identification of an optimal selection of autoregressive and moving average process parameters for the time series, including the seasonal component. The extended procedure involves an iterative approach, selecting parameters for the ARIMA model that minimises a given explanatory indicator of the model. Chakrabarti and Ghosh (2011) formulated a significant conclusion in time series model construction procedures, suggesting a departure from the arbitrary decision of the researcher regarding the selection of an information criterion due to the specific sensitivity of statistics to, among other things, the complexity of a given time series model. The researchers recommend selecting parameters for single-equation adaptive time series models using the Akaike Information Criterion (AIC) due to its forecasting properties (which is the subject of this study) and sensitivity to model complexity, as opposed to the Bayesian Information Criterion (BIC), which is resistant to model complexity. Information criteria are based on estimation using the maximum likelihood method that maximises the likelihood function, and empirical coherence is achieved by utilising the logarithm of the likelihood function, where $\ell = \ln(\mathcal{L})$. The AIC for the selected model is estimated as follows:

$$AIC = -2\frac{\ell}{N} + 2\frac{K}{N}$$

where:

N – number of observations,

K – number of model variables,

ℓ – logarithm of the likelihood function.

An elementary condition for optimising the ARIMA model and estimating it through the maximum likelihood method is the assumption of a Gaussian process for the residual component of the ARIMA model. Therefore, the estimation of the variance of the random component is obtained from the following equation:

$$-2\frac{\ell}{N} = const + \ln(\tilde{\sigma}_\varepsilon^2)$$

$$AIC = \ln(\tilde{\sigma}_\varepsilon^2) + 2\frac{K}{N}$$

A popular indicator used to assess the accuracy of ex post forecasts of a selected model is the root mean square error (RMSE). The RMSE indicates the average deviation of empirical values conditioned on the information sequence in the time series from the expected forecast values by the model. The estimation of the root mean square error is calculated by:

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\varepsilon_t^2}$$

Of course, in the literature, one can find many methods for estimating forecast errors and assessing the predictive ability of the model, including, among others, mean absolute error of ex post forecast (MAE), mean absolute percentage error (MAPE), and Theil's coefficient (Dmytrów & Doszyń, 2014).

The final stage of the procedure involves testing the hypothesis of a Gaussian distribution of the estimated model's residual component and the autocorrelation effect. To test the hypothesis of a Gaussian probability distribution of the residual component, the Jarque-Bera goodness-of-fit test will be conducted, which is based on the $\chi 2$ distribution with 2 degrees of freedom, skewness (third central moment) and kurtosis of the distribution (fourth central moment). The JB test is designed to examine the Gaussian nature of the residual component in single-equation models (Domański, 2010), including ARIMA class models. This is due to the test's sensitivity to the complexity of the N-dimensional residual component of multivariate time series models. The JB test (H_0: $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$) is conducted by estimating:

$$JB = \frac{N}{6}\left(\frac{\sum_{t=1}^{N}u_t^3}{N}\right)^2 + \frac{N}{24}\left(\frac{\sum_{t=1}^{N}u_t^4}{N}\right)^2$$

The rejection of the null hypothesis about the presence of a Gaussian residual component process, known as the leptokurtosis effect, most commonly results from the nonlinearity of the time series process, influenced by asymmetric information, leverage effects and variance clustering effects (e.g. the ARCH effect). In such a situation, SARIMA models require further optimisation procedures to incorporate the synthesis of conditional heteroskedasticity processes, such as ARIMA-GARCH models (Zhou et al., 2006). The realisation of a white noise stochastic residual component process determines the elementary condition of the absence of autocorrelation. Testing the hypothesis of autocorrelation is done using a portmanteau test; in this work, the Ljung-Box test statistic based on the $\chi 2$ distribution will be applied:

$$LB = N(N+2)\sum_{k=1}^{K}\frac{r_k^2}{N-k}$$

In the case of a Gaussian process integrated of order zero[8], for the residual component of the ARIMA model (r_k), the null hypothesis assumes values for $k \in 1, 2, \ldots, K$.

3. Results

The author has arbitrarily chosen a confidence interval for hypothesis testing with a significance level $\alpha = 0.05$. Significance levels for the estimation procedure are denoted by "*", such that significance levels are indicated as "** ≤ 0.01", "** ≤ 0.05", "* ≤ 0.1". The conducted tests and results of the estimation of constructed models were prepared using available libraries in Rstudio.

The generated time series plots (see Figure 2) for price dynamics, seasonal component and first differences allow for an initial assessment of stationarity. However, testing the hypothesis of stationarity requires conducting appropriate tests. After performing an extended Augmented Dickey-Fuller (ADF) test for the presence of a unit root, the results of statistics for the time series of Consumer Price Index (CPI) dynamics (–0.1544) and the seasonal component (–2.1081**) showed the absence of a unit root for the seasonal component. However, after conducting the KPSS test, both time series showed non-stationarity (for price dynamics – 1.32*** and the seasonal component – 1.049***). Following the ADF test for the CPI dynamics time series (–5.94***) and the seasonal component (–6.087***), the statistics indicated the absence of a unit root – stationarity of the time series in a broader sense. Accordingly, the integration parameter is assumed to be $d = 1$ and $D = 1$ (for seasonal component).

The initial selection of autoregression and moving average process parameters for the first difference time series, according to the classical Box-Jenkins procedure, showed statistical significance for the parameters; $p = 1$, $q = \{0,1,2,3\}$ (see Figure 3 and Table 2).

In the case of the seasonal component, optimal parameter selection according to the classical procedure indicates: $P = \{1,2\}$ and $Q = \{0,1,2,3\}$ (see Table 3).

[8] A Gaussian process integrated of order zero is assumed for Y_t:

$$Y_t \sim N(0, \sigma_t^2)$$
$$Y_t \sim I(0)$$

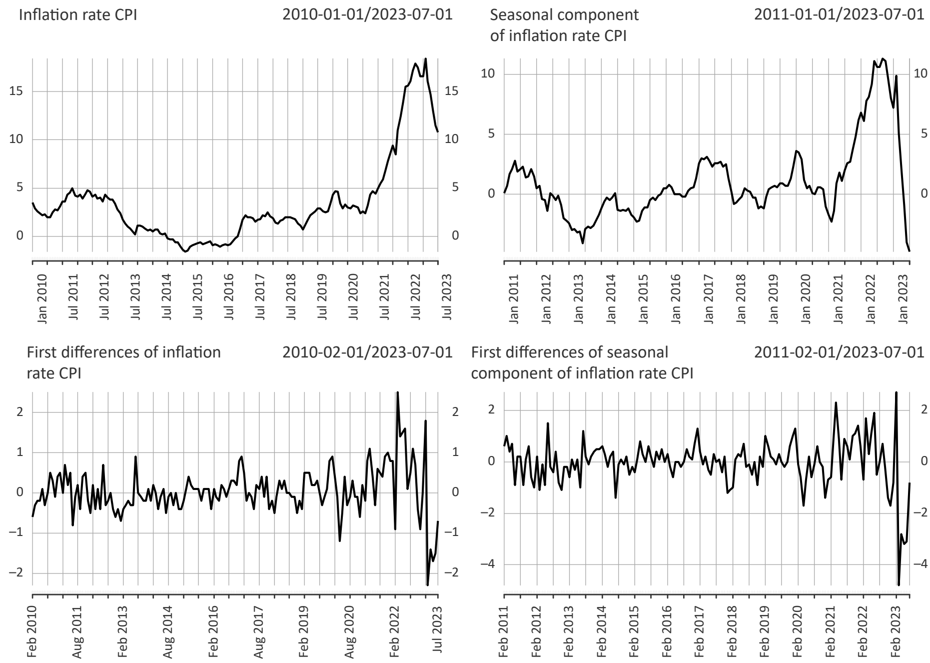

Figure 2. Time series charts; dynamics of CPI and seasonal component along with their first differences

Source: own elaboration.

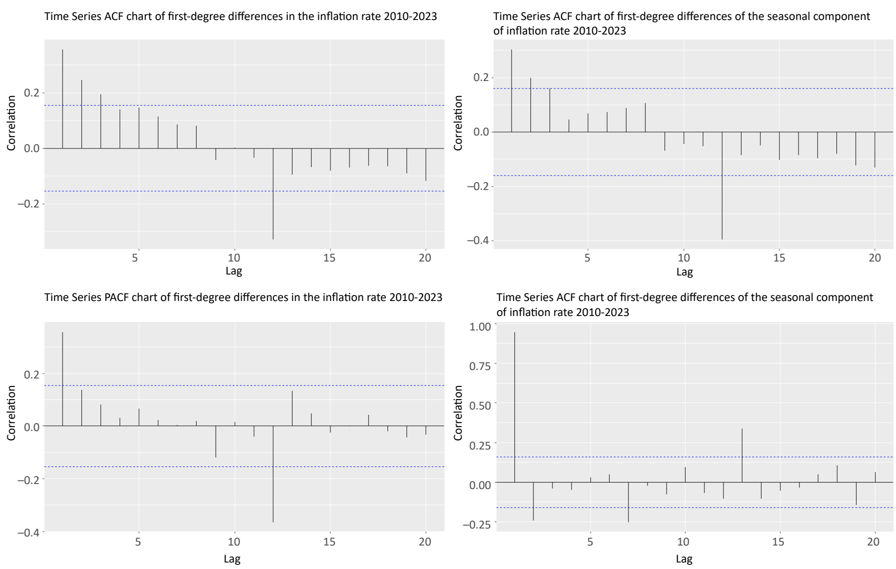

Figure 3. Autocorrelation charts and partial autocorrelation charts of selected components' processes

Source: own elaboration.

Table 2. Autocorrelation function (ACF) and partial autocorrelation function (PACF) statistics for selected lags of time series of CPI first differences

Lag	PACF	Significance	Lag	ACF	Significance
1	0.356736711031677	*	0	1	*
2	0.136288373134338	–	1	0.356736711031677	*
3	0.0818348962699699	–	2	0.246205248439543	*
4	0.0309894557029078	–	3	0.194573144706838	*
5	0.0654148792313358	–	4	0.14068731473643	–
6	0.0220859225688047	–	5	0.146903050935267	–
7	0.00391326094689732	–	6	0.115106817856936	–
8	0.0193427687124015	–	7	0.08513534408966	–
9	–0.119093117231304	–	8	0.081570634360881	–
10	0.0160967796089348	–	9	–0.0414943618958088	–

Note: * indicates statistical significance.

Source: calculations carried out using Rstudio.

Table 3. Autocorrelation function (ACF) and partial autocorrelation function (PACF) statistics for selected lags of time series of CPI first differences of seasonal component

Lag	PACF	Significance	Lag	ACF	Significance
1	0.94722613529003	*	0	1	*
2	–0.241023924223004	*	1	0.304848924414357	*
3	–0.0396112161986772	–	2	0.19892884491104	*
4	–0.0468058372979025	–	3	0.160870204960376	*
5	0.0300364579425894	–	4	0.0463530127322375	–
6	0.0507735091284552	–	5	0.0690582444834766	–
7	–0.251364686585908	*	6	0.0731551560011233	–
8	–0.0232292345056342	–	7	0.0881495591938981	–
9	–0.0759147182405396	–	8	0.107249735009197	–
10	0.0933647933626355	–	9	–0.0686675152624796	–

Note: * indicates statistical significance.

Source: calculations carried out using Rstudio.

According to the specified extended procedure, the optimal linear combination of autoregressive and moving average process parameters will be estimated using an iterative method based on the AIC information criterion for the first differences of time series.

The estimation showed the best empirical coherence of the model with empirical data based on the information derived from the time series for ARIMA(1,1,2) (see Table 4).

Table 4. Akaike Information Criterion (AIC) statistics for autoregressive and moving average process parameters for the time series dynamics of CPI prices

	***q* = 0**	***q* = 1**	***q* = 2**	***q* = 3**
***p* = 0**	290.949	275.8887	272.8257	271.9058
***p* = 1**	270.0245	265.532	267.4238	269.2545
***p* = 2**	268.3041	267.4356	265.4293	267.4223
***p* = 3**	268.5437	269.2918	267.4222	264.1817

Source: calculations carried out using Rstudio.

The iterative estimation of the AIC criterion for the seasonal component showed the best fit to the data for the models SARIMA(1,1,2)(2,1,2)12 and SARIMA(1,1,2)(2,1,3)12 (see Table 5). According to the above results, these models are optimal in terms of the extended procedure and will be used to construct short-term conditional forecasts.

Table 5. Akaike Information Criterion (AIC) statistics for autoregressive and moving average process parameters for the time series of seasonal component of dynamics of CPI prices

	***q* = 0**	***q* = 1**	***q* = 2**	***q* = 3**
***p* = 0**	404.0197	394.3976	393.5301	391.737
***p* = 1**	391.2828	389.2711	391.2365	393.629
***p* = 2**	390.7691	391.2394	386.6657	388.2564
***p* = 3**	391.205	392.9151	388.1219	385.7171

Source: calculations carried out using Rstudio.

The results of the estimation of SARIMA(1,1,2)(2,1,3)12 models showed a better fit for the SARIMA(1,1,2)(2,1,3)12 model (see Table 6). This is indicated by a lower value of the Akaike Information Criterion (AIC) and a higher likelihood ratio value.

The generated unit circles (Nazarko & Chodakowska, 2022) for selected SARIMA models indicate that the roots of the characteristic equation of autoregressive and moving average process parameters are inside and on the complex plane (see Figure 4), thus both models are stationary (Dritsaki, et al., 2021) (stable) and invertible.

The constructed fan chart (see Figure 5) for both models for the forecast period August 2023 – July2024 indicated deflationary tendencies. According to the SARIMA(1,1,2)(2,1,2)12 model, a return to the implementation of the inflation targeting strategy by the Monetary Policy Council's decision-making body is expected

Table 6. Results of estimating optimal (according to the extended Box-Jenkins procedure) SARIMA models

		$SARIMA(1,1,2)(2,1,2)_{12}$	$SARIMA(1,1,2)(2,1,3)_{12}$
d = 1	*p* = 1	0.201 (0.746)	–0.134 (0.392)
	p = 2	–	–
	q = 1	–0.425 (1.495)	–0.010 (0.657)
	q = 2	–0.092 (0.832)	–0.271 (0.324)
D = 1	*P* = 1	–0.021 (1.643)	–0.978 (0.075)
	P = 2	0.047 (0.289)	–0.802 (0.066)
	Q = 1	–0.425 (1.498)	0.503 (0.596)
	Q = 2	–0.092 (0.832)	0.460 (0.614)
	Q = 3	–	–0.525 (0.595)
AIC		282.01	271.69
Likelihood ratio		–133	–126.85

Source: calculations carried out using Rstudio.

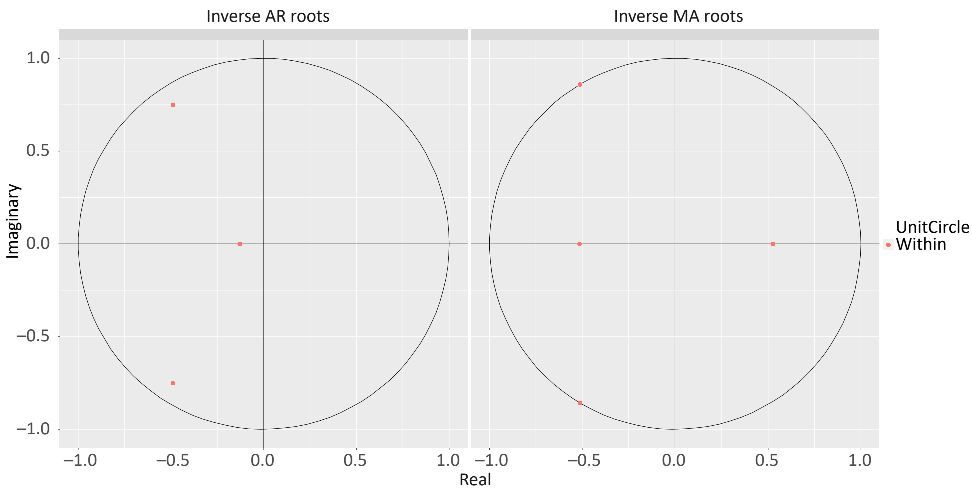

Figure 4. Unit circle plots for the optimal parameters of SARIMA models

Source: own elaboration.

in February 2024, while for the SARIMA(1,1,2)(2,1,3)12 model, it is expected in January 2024. Table 7 shows the exact point forecast with 95% confidence interval.

The ex post forecast accuracy assessment based on the root mean square error (RMSE) estimator indicates a smaller systematic bias (stronger coherence with em-

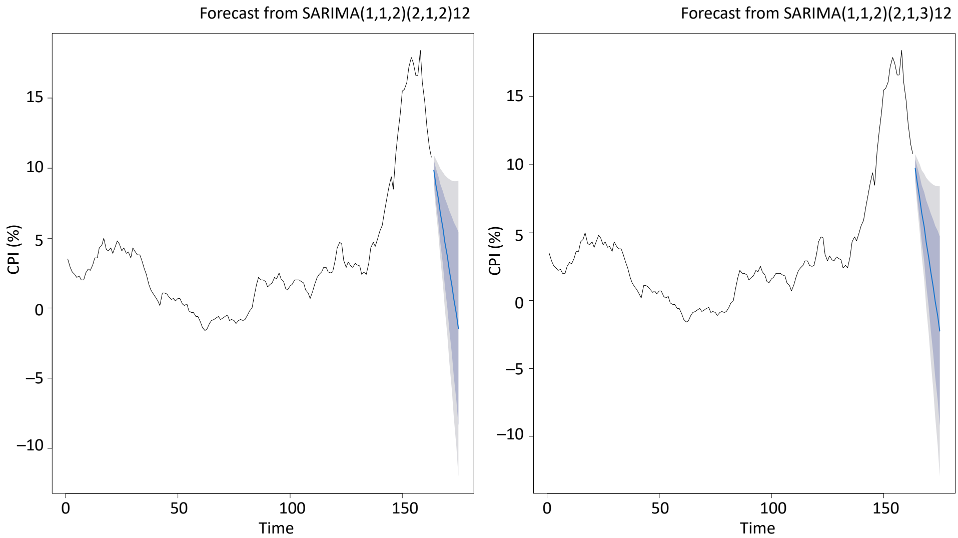

Figure 5. Conditional forecast (projection) of CPI price dynamics for the period August 2023–July 2024 (in monthly sequence) using optimal SARIMA models

Source: own elaboration.

Table 7. Expected values for the forecast period (point estimate and interval estimate with a 95% confidence interval) estimated using optimal SARIMA models

SARIMA(1,1,2)(2,1,2)$_{12}$			SARIMA(1,1,2)(2,1,3)$_{12}$		
Point Forecast	**Lo 95**	**Hi 95**	**Point Forecast**	**Lo 95**	**Hi 95**
9.843662341	8.762592	10.92473	9.745816077	8.714403	10.77723
8.823335356	7.023673	10.623	8.693728448	6.937199	10.45026
7.794948859	5.306436	10.28346	7.541947745	5.065332	10.01856
6.767388709	3.561343	9.973434	6.440060713	3.286568	9.593553
5.740168176	1.770953	9.709383	5.419793454	1.518554	9.321033
4.713126937	–0.06694	9.493192	4.273083439	–0.49974	9.045909
3.686127531	–1.95123	9.323482	3.185492976	–2.43392	8.804908
2.659141617	–3.88	9.198285	2.141331493	–4.39045	8.673114
1.63215858	–5.85137	9.115687	1.007297191	–6.51867	8.533265
0.605176356	–7.86359	9.073945	–0.073635778	–8.5896	8.442329
–0.421805702	–9.91511	9.071495	–1.134460416	–10.699	8.430052
–1.448787715	–12.0045	9.106925	–2.257532488	–12.9292	8.414181

Source: calculations carried out using Rstudio.

pirical data) for the SARIMA(1,1,2)(2,1,3)12 model (RMSE = 0.52) compared to the SARIMA(1,1,2)(2,1,2)12 model (RMSE = 0.55). Accordingly, the higher accuracy of the short-term conditional forecast tool using the SARIMA(1,1,2)(2,1,3)12 model allows for shaping forward guidance policy with greater confidence by influencing expectations channels. It serves as an optimal tool for constructing a conditional short-term forecast, and among the selected optimal models with strong empirical coherence, it enables the construction of the probability density function for the expected CPI dynamics.

In the further diagnostic procedure of short-term conditional forecast model optimality, the SARIMA(1,1,2)(2,1,3)12 model will be utilised, as it exhibited a lower bias of the estimator after estimating the results of the root mean square error ex post (RMSE) for both SARIMA models.

The generated charts (see Figure 6), including Quantile-Quantile plots, Gaussian probability density functions along with histograms (frequency distributions adjusted using the Freedman-Diaconis technique), Epanechnikov kernel density estimator, as well as theoretical and empirical cumulative distribution functions of the Gaussian distribution, clearly show the intensity of the dispersion of outlier values in the tails of the distribution. This is a result of the leptokurtosis of the residual component in the SARIMA(1,1,2)(2,1,3)12 model. After conducting the

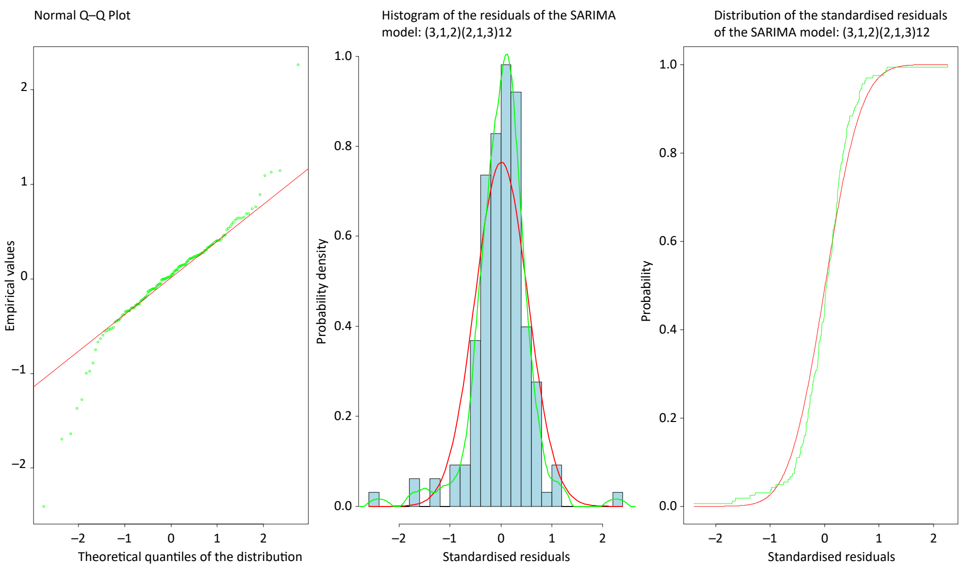

Figure 6. Charts: probability density distribution, quantile-quantile plot and cumulative distribution function plots for the residual component of the SARIMA(1,1,2)(2,1,3)$_{12}$ model

Source: own elaboration.

Jarque-Bera goodness-of-fit test for the univariate residual component of the SARIMA(1,1,2)(2,1,3)12 time series model, the alternative hypothesis of a distribution different from normal for the residual component should be accepted (172.68***). Additionally, testing for kurtosis revealed leptokurtosis (4.9), and skewness indicated left-skewness (–0.59). According to the developed diagnostic procedure, there is a stochastic process with stronger empirical coherence than the process based on the SARIMA(1,1,2)(2,1,3)12 model. The leptokurtic distribution of the residual component may arise from the nonlinearity of the time series, suggesting an extension of the model to a combination of SARIMA and GARCH-class conditional heteroscedasticity models (Pahlavani & Roshan, 2015) to reduce systematic bias through the identification of variability (conditional variance) of the residual component. Baciu (2015) makes a similar point about the process of conditional heteroskedasticity, identifying the non-Gaussian process of the residual component of the constructed ARIMA model.

Table 8. Autocorrelation process statistics for the residual component of the SARIMA(1,1,2)(2,1,3)12 model

Lag	Ljung-Box	*p*-value
1	0.000671489	0.979327
2	0.066421	0.967335
3	0.130059343	0.988001
4	0.805266847	0.937741
5	1.035684448	0.959637
6	1.775718802	0.939127
7	2.334342314	0.939046
8	2.513942771	0.961078
9	2.631238116	0.977143
10	2.750996158	0.986692

Source: calculations carried out using Rstudio.

The SARIMA(1,1,2)(2,1,3)12 model, at the stage of diagnosing the autocorrelation process of the residual component, exhibited a lack of persistence after conducting the portmanteau test (see Table 8). For the included lags of the residual component, the null hypothesis of no autocorrelation process should be accepted.

Conclusion

Although the conducted stage of diagnosing the optimal model, following the extended procedure, reduced the model's reliability stemming from the leptokurtosis effect of the residual component, the SARIMA(1,1,2)(2,1,3)12 model for the examined time series spanning from January 2010 to July 2023 remains a high-quality model conditioned by values of explanatory indicators, ex post forecast accuracy and lack of persistence (lack of autocorrelation process in the residual component of the model). Furthermore, the results of short-term conditional forecasting, considering the current statistics from Statistics Poland regarding the relative values of the CPI dynamics indicator for the periods from August 2023 to November 2023, exhibited accuracy within an arbitrarily set 95% confidence interval.

An analogous procedure for identifying the orthogonal parameters of the optimal SARIMA model for the construction of a conditional forecast of the rate of change in prices as measured by the CPI was used for the economies of the Philippines (Corpin et al., 2023), Turkey (Şanlı & Özmen, 2017) and Ghana (Havi, 2023), among others. The developed procedure allowed the selection of an optimal model with low values of explanatory indices and ex-post forecast accuracy ratings, which defines the SARIMA model as a reliable tool for building short-term conditional forecasts of price dynamics for the analytical background of central banks.

An interesting application of SARIMA models to the construction of a conditional forecast of core inflation in the Ukrainian economy is presented by Krukovets (2024). The author, conducting validation and evaluation of the predictive ability of selected models, includes the form of the SARIMA model and a version that is a combination of the form of the model along with the integration of the neural network machine learning technique (LSTM) for the process of the residual component of the optimal SARIMA model. The results of the estimation of the integrated SARIMA model with the LSTM machine learning technique show that the model has the best fit to the empirical data among the tested models.

The constructed SARIMA model is a model with short memory, as a single-equation time series model with strong empirical coherence that does not include variables that, from the perspective of a given theory (calibration incorporating rational expectations and microeconomic foundations) or empirically conditioned coherence and structural examination of causality, would allow for the identification of structural shocks and the resulting price adjustment processes. The SARIMA model is a short-term conditional forecasting model, and therefore, the stochastic process allows for constructing forecasts based on information obtained in the optimisation procedure (reduction of estimator bias) of orthogonal parameters of the SARIMA model.

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