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Application of the vector-autoregression VAR model in the analysis of unemployment hysteresis in the context of Okun's Law

Patryk Kołbyko[1]

Abstract

Unemployment is an important macroeconomic issue both in theoretical terms and for economic reality. On the theoretical ground, the unemployment rate, which is a measure of the share of unemployed units of the labour supply in the economy, determines the output gap at a certain adjustment parameter determined by the marginal productivity of labour. One of the causes of rising or persistent unemployment in the economy is the phenomenon of unemployment hysteresis, which occurs as a result of changes in the marginal disutility of labour, the strength of the wage bargain and other exogenous conditions arising in previous periods. The purpose of the study conducted in the following paper is to investigate the phenomenon of hysteresis in the labour market by analysing the significance of the impact of the unemployment rate in previous periods. In addition, the work aims to study Okun's Law as an effect of production dynamics on the unemployment rate. The study of the dependence was carried out through the estimation of a macroeconometric time series model—vector-autoregression (VAR) on the example of statistical data for Poland obtained from Statistics Poland (Stat.gov.pl) and complied raports about national accounts in the quarterly sequence for the years 2015–2021. The period of the study was

Keywords

- vector-autoregression model VAR
- time series analysis
- hysteresis in the labour market
- Okun's Law
- macroeconometrics

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arbitrarily selected with the observation of business cycle fluctuations in the above time frame. Empirical analysis of selected structural parameters through estimation of the vector-autoregression model showed a significant influence of the time series in the formation of the unemployment rate, which confirms the influence of the analysed phenomenon of hysteresis in the labour market. In addition, the vector-autoregression model for interval forecasting through the use of dynamic prediction proved to be a posteriori accurate forecasting model of the unemployment rate in the Polish economy.

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Introduction

In the literature of economic theory, there are many concepts from other sciences that have been implemented for the purpose of describing phenomena, which by the keen observation of researchers have been formulated to detail a certain mechanism occurring in economic life. Many economic methodologists to this day argue about the validity of using certain tools of methodology to study certain phenomena; some use tools specific to the methodological field of positive economics (Friedman, 1966), that is, the use of estimation by quantitative methods and stochastic processes to objectively evaluate economic phenomena; others use broad description using deductive or inductive approaches, and still others synthesise the above methods taking into account certain normative assumptions *a priori* or *a posteriori* using non-linear models. One phenomenon that has attracted the attention of a wide range of empirical researchers and macroeconomists since the second half of the 20th century is unemployment hysteresis. The concept of hysteresis, which derives its name from the Greek *hústeros* ("to lag behind") was originally coined by Scottish physicist Alfred James Ewing, who used the term hysteresis to describe the permanent effect of the temporary influence of ferrous metals on magnetic fields. A general description of the phenomenon of hysteresis was formulated more broadly within the framework of systems theory by Mark Krasnoselsky and Alexei Pokrovskii in *Systems with hysteresis* (Cross et al., 2005), who pointed out the nonlinearity in the way elements respond to changes in the system. In line with the above, the phenomenon of hysteresis is also a platform for research in chaos theory, where hysteresis can have a significant impact on fluctuations in the limit cycle (Cavallo et al., 2005) as the equilibrium point to which the system is moving while in transition. More widely echoed

in economic theory was Pokrovskii, who analysed business cycles with reference to Preisach's hysteresis model (Mc Namara & Pokrovskii, 2006) as an implication to Kaldor's (1940) endogenous limit cycle model. In terms of endogenous growth models, which take into account the problem of quantity adjustment that is an implication of the acceleration mechanism (Clark, 1917) in the form of lags between the supply of capital goods and gross accumulation and investment spending and the marginal efficiency of capital, time is an important factor that determines the volume of the dependent variable under study in the future period. An example of a theoretical growth model that uses the acceleration mechanism of the acceleration rule in the time interval of the dependent variable under study is the endogenous Samuelson-Hicks model (Bohner et al., 2010). In the "static" understanding of economic mechanisms, hysteresis is a phenomenon resulting from external factors such as psychology, which is revealed when the change in the reservation wage (Karunaratne, 1995) is determined by the marginal disutility of labour. A change in the psychology of workers, a change in labour laws, the strength of union labour bargaining and minimum wages are exogenous factors that determine the rate of structural unemployment over a certain time period (Guichard & Rusticelii, 2010), where the economic sense of the unemployment hysteresis phenomenon is revealed. Another example of the hysteresis phenomenon is when the employment status of workers is determined by the bargaining power over the wages of currently employed workers, who determine the propensity of the private sector to hire workers at set wages, where in the time interval studied by the influence of an external factor, the unemployment rate is formed, such a hysteresis phenomenon is referred to as "insider-outsider" (Gali, 2020; Grinfeld et al., 2009).

Okun's law, which is a rule that was originally derived from the statistical relationship between the unemployment rate and the output gap, which was estimated by A. Okun (1962), is also the subject of research into the relationship between the unemployment rate and output dynamics. The above law in terms of the new neoclassical synthesis takes into account in its analysis the natural unemployment rate and the adjustment parameter determined by the marginal productivity of labour as the ratio of the growth of labour product to the growth of employed labour factors calculated in wage units. In practice, however, with a mesocurtic (Batóg & Batóg, 2012) distribution of marginal workers' labour product (productivity), marginal labour productivity must exhibit a value greater than zero. This is due to the fact that each employed labour factor at a given disutility of labour shows a certain marginal product of labour (Zwiech, 2013) under conditions of output gap. Of course, under conditions of inflationary disequilibrium caused by the impact of the income effect, for example, under the fiscal stimulus applied or the overestimated level of the marginal efficiency of capital, will actually cause a non-linear price increase under price adjustment, but also at a much

slower rate of quantity adjustment resulting from the principle of capital adjustment. Okun's work, which is an empirical analysis, showed—using US statistics as an example—a relationship in which a 1% change in output dynamics determined a 0.3 p.p. change in the unemployment rate. By many researchers, the above relationship with the confidence interval taken into account has been empirically confirmed on the example of other economies (Lee & Huruta, 2019), sometimes examining the above relationship with extended vector-autoregression models (Pata et al., 2018).

The purpose of the study discussed in the paper is to analyse the relationship between the unemployment rate in the Polish economy and the time series and production dynamics using the estimation of a macroeconometric vector-autoregression VAR model. The work is designed to demonstrate the existence of the phenomenon of hysteresis in the labour market through the relevance of the impact of the state of employment in the studied time series and the relationship derived from the economic theory of Okun's Law. In addition, in order to empirically verify the VAR model, a static forecast was carried out for the estimated trend model used in the dynamic forecast of the vector-autoregression model to show "drift" in random straying of hysteresis phenomenon. The methodology used in the paper applies the stochastic approach within the positive field of economics to reliably estimate and objectively draw conclusions from the analysis while synthesising the inductive approach characteristic of the normative field of economics, where economic theory is relegated to the role of a "scenery" used to describe empirical research, and the estimated model was drawn up by meeting assumptions about the reliability of model verification and the optimal choice of structural parameters and their lags. One of the precursors of this approach was the American representative of institutionalism Wesley Mitchell (Rutherford, 1987). The compiled data on quarterly unemployment rate and production dynamics for empirical estimation are secondary data obtained from the compiled report for Statistics Poland on the national accounts in 2015–2021 (Jeznach, 2017; Perzyna, 2022).

1. Literature review

The vector-autoregressive VAR model developed and proposed by C. A. Sims in 1980 in his work Macroeconomics and Reality provides an alternative macroeconometric model to the multilinear models, the fundamentals of construction and estimation of which were presented by the Cowles Commission. Multiequation macroeconometric models imply the problem of identifiability (Wójcik,

2014) of the studied structural parameters of the estimated model. The structural parameters of the examined multivariate model based on economic theory are subjectively fitted, so that if the examined econometric model lacks identifiability, its reduced form should be changed by adding or removing exogenous variables. The choice of selected regressors in the model is done by subjectively and arbitrarily selecting variables based on economic theory and estimating them by removing quasi-constant variables using the coefficient of variation or examining the correlation between the variables under study (dependent and independent) and multiple correlation. In Poland, an example of a hybrid multivariate model used by the National Bank of Poland to project inflation and output dynamics is the NECMOD model (Budnik et al., 2009), which takes into account, in accordance with classical theory, supply-side factors and the Keynesian implication in the form of fiscal stimulus. The above model, therefore, ranks between the mainly theoretical classical macroeconometric DSGE model and the vector-autoregression VAR model, which fully takes into account only empirical data. The exemplary multivariate NECMOD model ranks among the hybrid models, as shown in Figure 1.

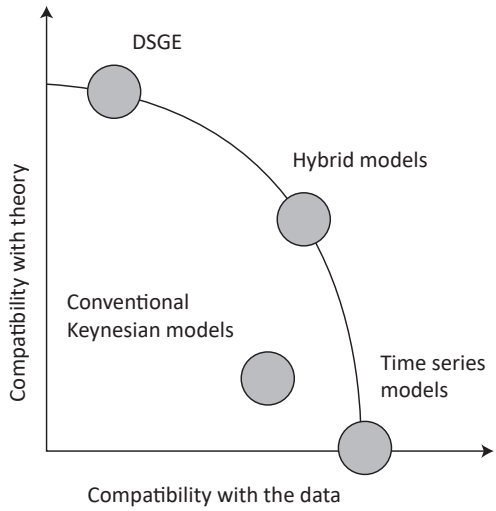

Figure 1. Typology of models used by analytical facilities of central banks

Source: own elaboration based on: Hara et al. (2009).

In the literature, the macroeconometric VAR model is a reliable (mainly by its consistency with empirical data) model for estimating Okun's Law as the relationship of output dynamics and the unemployment rate in the time series under

study, indicating the significance of the effect of lags on the variable under study. In addition, the model itself is also used especially for forecasting short-term economic activity, price dynamics (Robinson, 1998) or studying the impact of supply and demand shocks (Blanchard & Quah, 1988) on the above variables.

Representatives of the current of neoclassical synthesis use the concept of the phenomenon of hysteresis in the labour market as a sometimes exogenous cause of the occurrence of the production gap resulting from the rate of unemployable factors of production, as it results from a psychological factor in the form of disutility of labour. Hysteresis as a phenomenon in the deterministic view is the result of a traversal of a stationary state at a constant value of endogenous structural parameters, so that in the systems theory view such a traversal would arise from exogenous factors. In stochastic process modeling terms, such an impact is determined by the random component if the regressor is not a variable that reliably determines the strength of workers' bargaining over wages resulting from the marginal disutility of labour (Jamróz & Kilon, 2015). Thus, in economic theory, such an exogenous factor can be referred to as a random stray with drift. The issue of the influence of worker pressure in wage bargaining as an exogenous determinant of the hysteresis phenomenon is presented by Blanchard and Summers (1986) in their analysis of the causes of unemployment in the 1980s in Europe. However, not only is the above case a determinant of unemployment arising from events in previous periods, such also include the time interval in the acceleration mechanism (Chenery, 1952). Also in the formation of the unemployment rate, the phenomenon of hysteresis influences through the exogenous impact of fiscal policy determining the fluctuation of disposable income of unemployable individuals in social transfers, thus creating a structural unemployment rate and causing an increasing surplus of marginal disutility of labour between successive periods.

2. Research methods

To analyse the phenomenon of hysteresis in the labour market as an impact of the time series on the unemployment rate in the economy, the VAR vector-autoregression model extended by the synthesis of the classical trend model will be used to implement the optimally selected (based on the acquired estimates) trend model.

One of the classical methods of studying the effect of the studied dependent variable of the model on time as a predictor is models for the trend—trend as time series models. An important assumption of the model is that the time vari-

able does not act as an endogenous factor in economic terms, but as an external, exogenous factor influencing the studied dependent variable in econometric terms for the time series model. Thus, it represents a predictor that synthesises the influence of variables not included in the reduced form of the model. The first step in examining the impact of the trend is to "smooth" the time series by determining the trend function. Thus, the theoretical notation of the form of the time series model can be defined as (Gajda, 2001):

$$Y_t = f(t) + \varepsilon_t$$

where: Y_t is dependent variable of the model; $f(t)$ is trend function, ε_t is random component of the model.

In the following section, using Klein's method, a model estimate will be made for the trend function as a predictor; linear, quadratic, cubic and logarithmic, as shown in the Table 1. Then the most reliable model will be selected taking into account the absence of autocorrelation of the random component of the model, the randomness of the residuals, the highest fitting coefficient $R2$ and the statistical significance of the tested independent variables of the model falling within the confidence interval of 5%.

Table 1. The forms of trend models

Trend	Model form
Linear	$Y_t = \alpha_0 + \alpha_1 t + \varepsilon_t$
Square	$Y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \varepsilon_t$
Cubic	$Y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \varepsilon_t$
Logarithmic	$Y_t = \alpha_0 + \alpha_1 lnt + \varepsilon_t$

Source: own elaboration.

The final step in the analysis of the hysteresis phenomenon for the studied predicates is the estimation of the vector-autoregressive VAR model. The VAR model proposed by C. A. Sims has the form (Kilian, 2011):

$$X_t = \sum_{i=1}^{k} A_i X_{t-i} + \varepsilon_t$$
$$t = 1, 2, \ldots, n,$$

where: k is row of the VAR model; X_t is vector of observations of population values of n variables of the model; A_i is matrix of autoregressive operators of indi-

vidual processes, in which it is a priori assumed that there are no zero elements; ε_t is vector of the residual component, in which the given components are correlated with each other but there is no significant autocorrelation of the residuals.

The modified VAR model, which is an extension of the Sims model, takes into account deterministic trend and seasonality:

$$X_t = A_0 D_t + \sum_{i=1}^{k} A_i Z_{t-i} + \varepsilon_t$$

where: A_0 is matrix of deterministic trend values; D_t is vector of observations of deterministic trend values.

The construction of the vector-autoregression model is, compared to multivariate models, quite simple in its construction, which facilitates the estimation of the model and does not require the theory necessary in the selection of exogenous variables as structural parameters of the model. However, the estimation of the model itself does not lead to any particular conclusion if the study of the impact of the time series does not take into account a predetermined motive in the form of a hypothesis underlying, for example, a macroeconomic rule or mechanism (such as unemployment hysteresis). In addition, in order to correctly estimate the model for the variable under study with a specified distribution of lags, it is necessary to verify the model:

- stationarity of the studied variables,
- optimal selection of the lag order of the model under study,
- the study of autocorrelation of the random component.

According to the included assumptions towards model verification, the stationarity of the model's residuals should first be examined using the Dickey-Fuller (ADF) test. The stationarity of the model requires that probability distributions with successive periods jointly and conditionally show no change with shifts, with weak stationarity of the time series being sufficient for the reliability of VAR model verification. Then, in order to avoid the undesirable phenomenon of autocorrelation, it is necessary to estimate the correlogram of structural parameters using the PACF partial autocorrelation test and the Ljung-Box test, which, together with the Akaike AIC, Schwarz BIC and Hannan-Quinn HQC information criterion tests, will allow to select the optimal lag distribution of the variables under study for the vector-autoregression model. After the estimation of the vector-autoregression model, the forecast will be estimated by using dynamic prediction, which will determine the point forecast with *ex ante* error and interval forecast.

The statistical data of the examined structural parameters of the estimated models were expressed as percentage change indexes. In addition, the estimation performed adopted a subjective confidence interval of $\alpha = 0.05$.

3. Results

The estimation of the trend model using Klein's method for the predictor under study showed an implication to the established assumptions. In the case of testing the randomness of the sample elements, the significance of the series test, where the *p*-value was 0.17, showed that the sample elements are random, so the established assumption of the randomness of the model residuals was met. The problem for the estimation of the trend model was the presence of significant autocorrelation of the model's 1st order residuals, so that in the time series studied, the variables showed autocorrelation of the residuals for the 1st order lag of the predictor. The best fit of the variables to the model showed the trend model for the quadratic trend, so that despite the presence of significant autocorrelation of the 1st degree residuals, the predictors of the time series model for the quadratic trend will be used. For the cubic trend model, the variable "cb_time" (cubic trend) showed a lack of statistical significance for the subjectively determined confidence interval. The estimation of the time series model for the trend for all forms of the model except the cubic trend showed significant dependence of the unemployment rate as a regressor of the model on the trend, as shown by the results in Table 2.

Table 2. Estimation of trend models; linear, quadratic, cubic and logarithmic for unemployment rate regressor

Series test result ≈ 0,17	**Model form**			
Dependent variable: UnemploymentRate	$Y_t = \alpha_0 + \alpha_1 t + \varepsilon_t$	$Y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \varepsilon_t$	$Y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \varepsilon_t$	$Y_t = \alpha_0 + \alpha_1 \ln t + \varepsilon_t$
Adjusted coefficient of determination R^2	72.88%	95.16%	95%	89.73%
p-value for α_0	≈0	≈0	≈0	≈0
p-value for α_1	≈0	≈0	≈0	≈0
p-value for α_2	–	≈0	0.039	–
p-value for α_3	–	–	0.65	–
Significance of Durbin Watson test statistic	≈0	≈0	≈0	≈0
The value of the Durbin Watson test statistic	0.22	1.07	1.06	0.47
Occurrence of significant autocorrelation of first-degree residuals	yes	yes	yes	yes

Source: calculations made with the Gretl statistical program using the Stat.gov.pl database (2022).

The next stage in the estimation of the time series model is the econometric forecast for the next two periods, taking into account the point and interval forecasts. The model used, in which the estimation was carried out using the classical method of least squares, takes the form of a quadratic trend, in view of which it is a forecast of the static type (Davidson & MacKinnon, 2004; Kufel, 2007). The extinct forecast for the predicate using the Gretl statistical program for the time series under study proved to be partially accurate. With ex-post errors taken into account, the point forecast estimates proved to be inaccurate for the periods: Q3 2015, Q1 2016, Q1 2017, Q3 2019, Q4 2020, Q1 2021 and Q4 2021. More relevant to the study of unemployment rate prediction, however, is the range forecast, as shown in Figure 2. Suggesting the acquired data of Statistics Poland on the registered unemployment rate, the estimation of the interval forecast of the quadratic trend proved accurate for all observations.

The first step in meeting the assumptions necessary for reliable estimation of the vector-autoregression VAR model is the optimal selection of the lag distribution

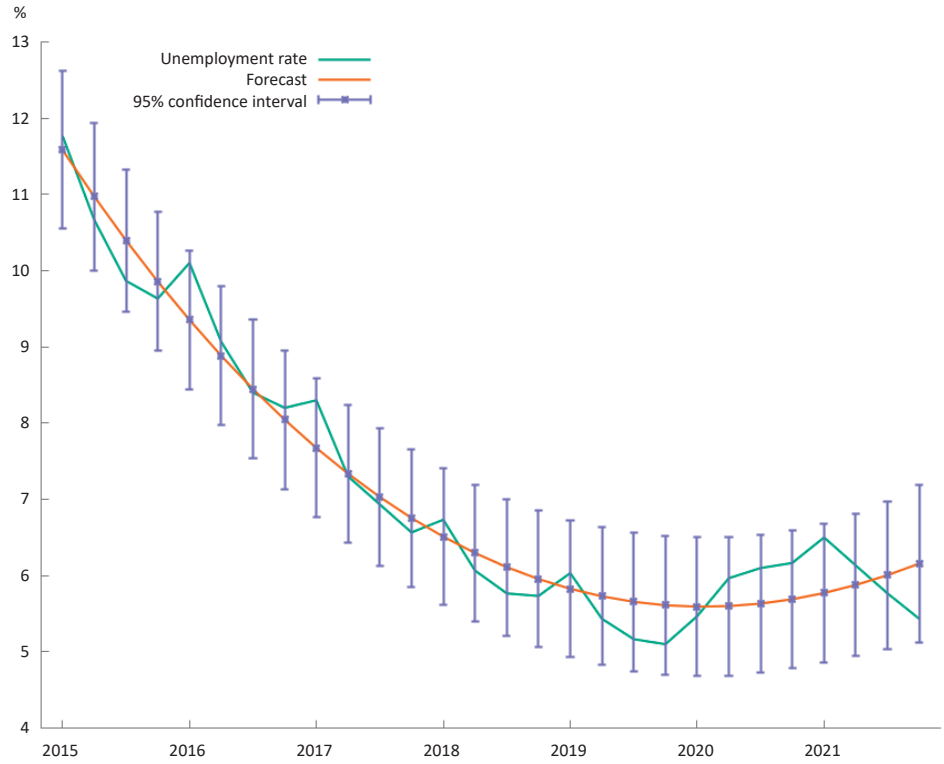

Figure 2. Graphical visualisation of the forecast of the quadratic trend model

Source: chart generated with the Gretl statistical program using the Stat.gov.pl database (2022).

of the structural parameters under study by estimating the Akaike AIC, Schwarz BIC and Hannan-Quinn HQC information criterion. In addition, to reliably verify the optimal selection of the lag distribution of the estimated VAR model, the reliability quotient LR will be considered (Hatemi-J & Hacker, 2009).

All the estimated information criteria in Table 3 indicate the selection of an order of delay equal to 3 as the smallest value of the information criterion. In addition, the LR reliability quotient for the delay order indicated as optimal by the criteria positively verifies the selection.

Table 3. Information criteria; AIC, BIC, HQC and LR reliability quotient test for constructed VAR model

Lag	loglik	*p*(LR)	AIC	BIC	HQC
1	–8.08626		1.340522	1.733207	1.444701
2	–3.63622	0.00285	1.053019	1.494789	1.17022
3	8.7446	0.00000	0.104617*	0.595473*	0.234841*

Note: * indicates the lowest value of the information criterion index.

Source: calculations made with the Gretl statistical program using the Stat.gov.pl database (2022).

For the ADF test, the variable of first differences in the unemployment rate will be used.

Conducting the ADF test showed that for a model without free expression, the null hypothesis should be rejected in favour of the alternative hypothesis, since the significance of the test conducted showed the lack of integration of the first degree, and therefore the stationarity of the time series in the model under study. According to the results of the test in Table 4, the first difference model of the variables should be used for the estimation of the VAR model.

Next, for the production dynamics variable, the ADF-GLS test was carried out, taking into account the significance of the lag from the indicated order of the modified AIC criterion and the Perron-Qu method (Perron & Qu, 2007), which showed

Table 4. Testing the stationarity of the random component of the model for the value of the variable and first differences

ADF test without free expression for first differences „UnemploymentRate"	
ADF test estimate	*p*-value
–4.015	≈ 0
ADF-GLS test without free expression for the variable "GDPgrowth"	
ADF-GLS test estimate	*p*-value
–2.168	0.03

Source: calculations made with the Gretl statistical program using the Stat.gov.pl database (2022).

statistical significance (*p*-value ≈ 0.03) for the values of the variables. According to the above study, the values of the production dynamics variable will be used to estimate the vector autoregression model and verify the model's assumptions.

The final step in the verification of variables as predictors of the dependent variable of the unemployment rate is the estimation of the correlogram of structural parameters using the PACF partial autocorrelation test and the Ljung-Box test, which will also allow to select the optimal lag distribution of the studied variables for the vector-autoregression model. Both the created variable for the first differences of observations of the unemployment rate showing stationarity of the time series and the production dynamics variable are not statistically significant for the third observation of the lag distribution for the PACF autocorrelation tests conducted, as shown in Table 5. For the Ljung-Box autocorrelation test, only the variable of the first increments of the unemployment rate did not show statistical significance.

Table 5. Test of autocorrelation with the PACF partial autocorrelation test and Ljung-Box for first differences in unemployment rate observations

	Delay	PACF autocorrelation test	Ljung-Box autocorrelation test	*p*-value
UnemploymentRate variable for first differences	1	0.1634	0.8039	0.37
	2	–0.1189	1.0523	0.591
	3	0.0126	1.0705	0.784
GDPgrowth	1	0.4367	5.9318	0.015
	2	0.1152	8.5365	0.014
	3	–0.2186	8.5436	0.036

Source: calculations made with the Gretl statistical program using the Stat.gov.pl database (2022).

The estimated vector-autoregression model, after taking into account the theoretical assumptions and the implications of the the cubic trend, which showed a very good fit of the variables to the unemployment rate regressor model estimate of the coefficient of determination *R*2 (95%), in the conducted study takes a reduced form:

$$dU_t = \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \beta_1 dU_{t-1} + \beta_2 dU_{t-2} + \beta_3 dU_{t-3} + \\ + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \gamma_3 Y_{t-3} + \varepsilon_t$$

where: dU is vector of observations of the variable „UnemploymentRate" for the first differences in the model; t, t^2, t^3 is vector of trend (linear, quadratic and cubic) of the model; Y is vector of the "GDPgrowth" predictor; α, β, γ is matrices of values of predictors of the model; ε_t is random component of the model.

The estimation of the model using Klein's method (Hall et al., 2014) showed that, with the confidence interval established, the distribution of third-degree lags was statistically significant for the "GDPgrowth" predictor for period $t-1$, the cubic trend, and all tested lags for the first differences of "UnemploymentRate", as shown in Table 6. Accordingly, the model positively verified the hypothesis of the existence of unemployment hysteresis in the Polish economy and, in accordance with the mechanism of the acceleration principle of the acceleration mechanism, the significance of production dynamics in previous periods. In addition, in the examined time series, the trend for the model showed high statistical significance, so that the phenomenon of hysteresis in the labour market in Poland defines random erring with drift determined by a deterministic trend. The coefficient of determination *R*2 for the model showed a satisfactory fit: 76.64% ∈ [60%;80%].

Table 6. Vector-autoregressive model estimation for selected predictors

Explained variable: dU_t	Coefficient	*p*-value
t	–0.328	≈0
t^2	0.03	≈0
t^3	–0.001	≈0
Y_{t-1}	–0.064	0.002
Y_{t-2}	–0.034	0.064
Y_{t-3}	–0.018	0.4692
dU_{t-1}	–0.865	≈0
dU_{t-2}	–0.8	≈0
dU_{t-3}	–0.778	≈0

Source: calculations made with the Gretl statistical program using the Stat.gov.pl database (2022).

The results shown in Table 7, show that the estimated point dynamic forecast for the estimated vector-autoregression model of the unemployment rate showed the model's lack of relevance to the actual unemployment rate during the period under study. The value achieved by the estimation was about 5.26% for Q1 2022 with an *ex ante* error of about 0.1788%, where the actual value of the unemployment rate in the quarter under study was 5.467%. However, the estimated dynam-

Table 7. Dynamic prediction for the predicate of the variable of first differences "UnemploymentRate"

Period	Point forecast	*Ex ante* error	Interval forecast	Actual value
1Q 2022	–0.173081	0.1788	[–0.554184;0.208022]	0.037

Source: calculations made with the Gretl statistical program using the Stat.gov.pl database (2022).

Figure 3. Graphical visualisation of the VAR model prediction for the dynamic forecast for the next period

Source: chart generated with Gretl statistical program using Stat.gov.pl database (2022).

ic forecast interval proved to be accurate, as the range of the estimated forecast value was ϵ [4.879146;5.641352], as visualized by the attached Figure 3.

4. Summary of the empirical research

After making the necessary assumptions to verify the reliability of the estimated time series models, which were then estimated, it may be concluded that the results produced using statistical inference met the analysis objective of the article. Both the model of the quadratic trend of the interval forecast and the vector-autoregression model using dynamic prediction method showed an accurate interval forecast in accordance with the actual data produced by Statistics Poland. In

addition, the estimation of the quadratic trend model showed the statistical significance of the time variables in the estimated model for the unemployment rate in the studied time series, so that the fluctuations in the state of employment are significantly determined by the trend in the studied time series. The estimation of the vector-autoregression model, the subject of which was the hysteresis in the labour market and the macroeconomic Okun's Law, showed that, except for the structural parameters for the third and second lags of the production dynamics, the examined structural parameters positively verified the phenomenon for the subjectively determined confidence interval.

Conclusions

The purpose of the study discussed here was to empirically verify the occurring phenomenon of hysteresis in the labour market and the relationship resulting from the macroeconomic Okun's Law. In order to carry out a reliable analysis, it was necessary to estimate structural parameters through statistical inference and use a macroeconometric time series model of vector-autoregression VAR synthesising the model with the trend of the unemployment rate. The study on the available statistical database for the period 2015–2021 in the quarterly sequence showed that the state of employment in Poland determines the time series, which then verifies the hypothesis of the occurrence of hysteresis in the labour market. In addition, the unemployment rate in Poland shows a significant dependence on the dynamics of production, so the study empirically verified Okun's Law positively.

The results obtained in the study after a reliable estimation of the vector-autoregression model and the included assumptions on the selection of the lag distribution and the stationarity of the time series showed that the dynamic forecast of the estimated model determined a posteriori accurate interval prediction comparing with the available data on the unemployment rate in Poland. Accordingly, the conducted study suggests the usefulness of the VAR model with consideration of the trend in forecasting the unemployment rate for the Polish economy. The application of the vector autoregression model also among other researchers on the example of the economy in the Philippines has proven to be a good model to study the hysteresis phenomenon when taking into account the impact of production dynamics (Valera & Dean, 2021).

The conducted analysis also provides a useful suggestion for analysts conducting research on economic fluctuations and policy mix practitioners to use time series models based to the greatest extent from the methodology of the field of positive economics characteristic of empiricism and economic operationism in place

of long-run macroeconometric models based on estimation under the assumption of stationary state of Walrasian equilibrium. Similar conclusions have also been reached by other econometricians, who have demonstrated the high utility and forecasting accuracy of the VAR model for the study of macroeconomic variables in Poland with the assumptions necessary for reliable model verification included in the following paper (Warzecha & Wójcik, 2014; Wójcik, 2014).

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