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A theoretical fundamental model of USD/CAD, gold spot and gold futures with interest rates, growth and financialisation

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Abstract

We develop a three-equation structural model linking the USD/CAD exchange rate, the spot price of gold and gold futures prices. The framework explicitly incorporates real interest rates and financialisation indices for both the United States and Canada, while output gaps influence the exchange rate indirectly through channels including monetary policy expectations, risk appetite and commodity demand. The model accounts for the commodity-currency nature of the Canadian dollar, the opportunity cost of holding gold, and the carry-arbitrage relationship augmented by speculative demand. We derive the model's reduced-form properties and discuss its key theoretical predictions as well as cross-equation feedback mechanisms. Preliminary empirical validation based on cointegration analysis and structural vector autoregression (SVAR) estimation confirms the model's principal theoretical predictions. The analysis highlights the amplification effects arising from the mutual feedback between the exchange rate and gold prices.

Keywords

- exchange rates
- gold
- commodity futures
- financialisation
- commodity currencies
- real interest rates
- USD/CAD
- cost of carry

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Introduction

The relationship between commodity currencies, precious metals and financial markets has become increasingly complex in an era of unconventional monetary policy, cross-border capital flows and financialisation of commodity futures (Cheng & Xiong, 2014; Tang & Xiong, 2012). The Canadian dollar (CAD) is a quintessential commodity currency (Cashin et al., 2002; Chen & Rogoff, 2003), yet its dynamics with gold – a global monetary asset – and the USD/CAD exchange rate cannot be fully captured by simple bivariate correlations. This paper develops a theoretical fundamental model that jointly determines the USD/CAD exchange rate, the spot price of gold and near-term gold futures prices. The model explicitly incorporates real interest rates, economic growth and a measure of financialisation in both the United States and Canada.

Motivation

Traditional exchange rate models (e.g. monetary approach, UIP) often fail to explain short- to medium-term movements in commodity currencies (Engel, 2016; Meese & Rogoff, 1983). Meanwhile, gold pricing models typically focus on US real rates and inflation expectations (Erb & Harvey, 2013; Frankel, 1986, 2008), ignoring the fact that gold is also a major Canadian export and therefore a driver of the CAD. Furthermore, the proliferation of exchange-traded funds (ETFs), algorithmic trading and speculative positioning in futures markets has introduced a “financialisation” channel that can temporarily decouple prices from macroeconomic fundamentals (Basak & Pavlova, 2016; Singleton, 2014). Our model aims to bridge these gaps by embedding gold as both a commodity (affecting Canada’s terms of trade) and a financial asset (sensitive to US real rates and speculative flows).

Endogenous variables

We define three key endogenous variables:

S_t – USD/CAD exchange rate (USD per CAD).
An increase denotes CAD appreciation.
G_t – spot price of gold in USD per fine ounce.
F_t – near-term gold futures price (e.g. 3-month) in USD per ounce.

The choice of F_t as the near-term futures price (defined as the nearest actively traded contract, typically the front-month contract with time to maturity T of approximately 1–3 months) captures the most liquid segment of the futures curve, where carry arbitrage and speculative positioning are most active (Fama & French, 1987; Pindyck, 1994). For empirical implementation, we use a constant maturity of $T = 3$ months (constructed via interpolation between adjacent contract months), which provides a fixed horizon for the cost-of-carry relationship.

Exogenous and policy variables

The model is driven by the following set of exogenous or policy-determined variables, measured for both the US and Canada where relevant:

The ex-ante real interest rate is defined as:

$$r_{US,t} = i_{US,t} - \pi^e_{US,t}$$
$$r_{CA,t} = i_{CA,t} - \pi^e_{CA,t}$$

where $i_{US,t}$ is the nominal yield on a 10-year US Treasury bond (or a 3-month Treasury bill, depending on the horizon), and $\pi^e_{US,t}$ is the expected inflation rate over the corresponding horizon. For empirical implementation:

1. For $i_{US,t}$ – use the 10-year US Treasury constant maturity yield or the 3-month Treasury Inflation-Protected Securities (TIPS) yield.
2. For $\pi^e_{US,t}$ – use the 5-year or 10-year breakeven inflation rate (defined as the difference between nominal and inflation-indexed Treasury yields) or by survey-based measures such as the University of Michigan 5–10 year inflation expectations or Consensus Economics forecasts.
3. The same approach applies to Canada using Government of Canada bond yields and survey-based inflation expectations.

$g_{US,t}$, $g_{CA,t}$: Output gaps (deviation of actual GDP from potential GDP, measured as a percentage of potential GDP). In empirical implementations where output gap data are not available at the required frequency, these may be proxied by growth momentum measures such as the year-over-year GDP growth rate or industrial production growth. While the baseline model excludes output gaps as direct regressors to maintain parsimony and avoid multicollinearity, they exert influence through three indirect channels: (i) monetary policy expectations affecting future interest rates, (ii) risk appetite and capital flows, and (iii) commodity demand. In an extended specification, the output gap differential $g_{US,t} - g_{CA,t}$ can be included directly.

Φ_t – financialisation index capturing ETF holdings, open interest and speculative positioning (e.g. CFTC COT data).

Risk_t – global risk aversion (e.g. VIX index or geopolitical risk indices).
$P_{oil,\,t}$ – price of oil in USD/barrel; included because oil is Canada's largest export.
Each variable plays a distinct theoretical role:

1. **Real interest rates** determine the opportunity cost of holding gold (for the US) and the carry trade incentive in USD/CAD (for both countries) (Lustig et al., 2011).
2. **Output gaps** capture cyclical demand pressures that may influence both monetary policy expectations and commodity demand.
3. **Financialisation** aggregates non-fundamental demand from institutional investors, trend-followers and ETF flows (Bhardwaj et al., 2013; Gorton & Rouwenhorst, 2006). This variable can drive gold prices independently of rates, and through the exchange rate, spill over into CAD.
4. **Risk aversion** affects both safe-haven demand for gold (Baur & Lucey, 2010; Baur & McDermott, 2010) and the USD's status as a reserve currency.
5. **Oil price** is a necessary control for CAD because Canada's trade balance is far more sensitive to oil than to gold (Amano & van Norden, 1995).

1. Literature review

Our work builds on three main strands of the literature: (i) commodity-currency exchange rate models, (ii) gold pricing models with a focus on real interest rates and safe-haven properties, and (iii) the theory of storage and financialisation in commodity futures markets. Each strand has made important contributions, yet they have largely developed in parallel, with limited cross-fertilisation between the currency, gold spot and futures dimensions. Our objective is to synthesise these perspectives into a unified framework that captures the simultaneous determination of the USD/CAD exchange rate, the spot price of gold and near-term gold futures, while explicitly accounting for the role of financialisation. To this end, for each literature strand we first summarise the key methodological approaches and empirical findings, then identify the specific gap that our model addresses. This structure allows us to highlight both the incremental extensions and the genuinely novel elements of our proposed framework.

1.1. Commodity currency models

Studies such as Cashin et al. (2002) and Chen and Rogoff (2003) link real exchange rates to commodity prices, establishing the empirical regularity that commodity-exporting countries' currencies appreciate when their main export prices rise. Methodologically, these studies typically employ cointegration analysis (Johansen, 1988) or panel data techniques to estimate the long-run relationship between exchange rates and commodity prices. The empirical findings consistently show that commodity prices are significant determinants of real exchange rates for commodity-exporting countries, with oil having a particularly large effect for Canada. Later extensions include Chen et al. (2010), who demonstrate that exchange rates can forecast commodity prices, and Ferraro et al. (2015), who examine the oil-USD/CAD relationship at higher frequencies. Lustig et al. (2011) further show that interest rate differentials and risk premiums are important drivers of currency returns, linking the exchange rate literature to broader asset pricing.

Research gap: existing models in this strand of the literature generally treat commodity prices as exogenous, without allowing for their simultaneous determination within a closed system. Furthermore, to the best of our knowledge, they do not fully incorporate financialisation spillovers within a structural framework.

1.2. Gold pricing models

Frankel (1986, 2008) emphasises real rates and inflation expectations, building on the Dornbusch (1976) overshooting model extended to commodities. The theoretical framework posits that commodity prices adjust to monetary policy shocks more quickly than goods prices, leading to overshooting. Baur and Lucey (2010) and Baur and McDermott (2010) examine safe-haven properties, using quantile regression and dynamic conditional correlation models to identify regimes where gold acts as a hedge (positively correlated with stocks in normal times) or a safe haven (negatively correlated during market turmoil). Erb and Harvey (2013, 2017) provide a comprehensive empirical assessment of gold's long-run fundamentals, finding that the ratio of gold to the US CPI is mean-reverting but with long half-lives, and that gold does not consistently hedge inflation in the short run.

Research gap: to the best of our knowledge, gold is not typically integrated into currency equations for commodity exporters in a fully simultaneous framework, despite its role in Canada's terms of trade. Moreover, the feedback from exchange rates to gold prices (operating through USD) is typically ignored.

1.3. Futures carry arbitrage and financialisation

The theory of storage (Pindyck, 1994, 2001) underpins futures pricing, linking spot and futures prices through the cost of carry, which includes interest rates, storage costs (c) and a convenience yield (y). Fama and French (1987, 1988) provide early empirical evidence on the forecasting power of futures prices and the role of the basis in predicting spot price movements. Pindyck and Rotemberg (1990) document excess co-movement of commodity prices beyond what fundamentals would predict, suggesting common factors or sentiment-driven dynamics. Gorton and Rouwenhorst (2006) document commodity futures performance, showing that commodity futures returns are positively correlated with inflation and negatively correlated with equity returns, making them attractive for portfolio diversification. Financialisation of commodity markets is analysed by Cheng and Xiong (2014), Tang and Xiong (2012), Singleton (2014), and Basak and Pavlova (2016), who show how index investors and hedge funds affect commodity prices through risk-sharing channels, limits to arbitrage and the rebalancing flows of commodity index funds. These studies demonstrate that financialisation can temporarily decouple commodity prices from traditional supply-demand fundamentals.

Research gap: to the best of our knowledge, existing models do not link futures financialisation premiums to spot exchange rates through explicit cross-market feedback mechanisms, nor do they allow cross-market feedbacks where futures speculation affects the spot exchange rate through the commodity-currency channel.

1.4. Contribution

The primary contribution of this paper is the development of a simultaneous three-equation structural framework that integrates the determination of the USD/CAD exchange rate, the spot price of gold, and gold futures prices within a unified system.

The novel features of the model are as follows:

1. It allows gold to enter the CAD equation as an endogenous variable (rather than exogenous, as in most commodity currency models).
2. It allows the CAD (via USD) to feed back into gold prices through an explicit exchange rate channel.
3. It introduces a financialisation premium in the futures market that is distinct from the cost of carry and directly linked to speculative positioning.

The model also extends existing approaches in several respects:

1. The CAD equation extends the standard commodity currency model (Chen & Rogoff, 2003) by using real rather than nominal interest rates and by treating commodity prices as jointly determined.

2. The gold equation extends the Frankel (1986, 2008) framework by explicitly incorporating financialisation and risk sentiment.
3. The futures equation extends the cost-of-carry model (Pindyck, 1994) by adding a speculative premium, following the insights of Cheng and Xiong (2014) and Singleton (2014).

This closed-loop system generates testable cross-equation restrictions and explains episodes where conventional fundamentals fail (e.g. the 2011 gold rally alongside USD strength).

2. Methodology

In this section, we develop the three structural equations that form the core of our theoretical model. The derivation proceeds step by step, beginning with the USD/CAD exchange rate equation, followed by the gold spot price equation, and concluding with the gold futures equation. Each equation is grounded in economic theory and incorporates the key channels identified in the literature review. We then assemble these equations into a simultaneous system and analyse its properties.

2.1. Equation 1: USD/CAD equilibrium

The Canadian dollar is widely recognised as a commodity currency. Its equilibrium value against the US dollar depends not only on financial factors (interest rates) but also on the real exchange rate implied by the terms of trade. We derive a reduced-form equation that combines uncovered interest parity with a commodity price channel, then augment it with a risk premium and cyclical adjustments.

2.1.1. Theoretical foundation

Let the real exchange rate be defined as

$$Q_t = S_t \frac{P_{US,t}}{P_{CA,t}}$$

where $P_{US,t}$ and $P_{CA,t}$ are price levels.

We employ the concept of a fundamental equilibrium exchange rate (FEER) – the exchange rate consistent with the economy being at full employment and sustainable external position. For a commodity-exporting country, the FEER is primarily determined by the terms of trade.

Following standard practice in the commodity currency literature, we focus on the terms-of-trade channel and abstract from relative productivity, which is typically slow-moving and can be captured by a constant (or a deterministic trend) in the empirical specification. Thus, we model the long-run equilibrium real exchange rate as:

$$Q_t^{LR} = \gamma_0 + \gamma_1 \frac{P_{gold,t}}{P_{US,t}} + \gamma_2 \frac{P_{oil,t}}{P_{US,t}}, \ \gamma_1, \gamma_2 > 0$$

The constant γ_0 captures, among other things, the long-run level of relative productivity.

The real exchange rate is linked to the nominal exchange rate by:

$$q_t = s_t + p_{US,t} - p_{CA,t}$$

where lowercase letters denote logarithms. Assuming that relative PPP holds for non-commodity goods in the long run ($p_{US,t} - p_{CA,t}$ is stationary), the nominal exchange rate s_t inherits the long-run properties of the real exchange rate. Thus:

$$s_t^{LR} = \gamma_0 + \gamma_1 g_t + \gamma_2 oil_t + \text{stationary terms}$$

where $g_t = \log G_t$ and $oil_t = \log P_{oil,t}$

2.1.2. Real interest rate parity and carry trade

Uncovered interest parity (UIP) states that the expected change in the nominal exchange rate equals the nominal interest differential:

$$E_t[\Delta s_{t+1}] = i_{US,t} - i_{CA,t} - \rho_t$$

where ρ_t is a risk premium.

The transition from nominal to real interest rates is justified by the Fisher equation:

$$i_{US,t} = r_{US,t} + \pi^e_{US,t}$$

and

$$i_{CA,t} = r_{CA,t} + \pi^e_{CA,t}$$

If PPP holds in the long run, the expected inflation differential is related to the expected change in the nominal exchange rate, and the UIP condition can be written in real terms:

$$E_t[\Delta q_{t+1}] = r_{US,t} - r_{CA,t} - \rho_t$$

Solving the model forward under the assumption that the real exchange rate mean-reverts to its long-run commodity-fundamental value q_t^{LR} yields:

$$S_t = \sum_{j=0}^{\infty} \frac{1}{(1+\theta)^j} E_t\left[(i_{US,t+j} - i_{CA,t+j}) - \rho_{t+j} - (q_{t+j}^{LR} - q_t^{LR}) \right] + \text{constant}$$

where θ is the speed of mean reversion.

Assumption 1. The forward solution is obtained under the following assumptions: (i) the real exchange rate q_t is stationary and mean-reverts to its long-run fundamental equilibrium q_t^{LR}; (ii) the speed of mean reversion θ is constant and positive; (iii) expectations are formed rationally; and (iv) the transversality condition holds, so that explosive solutions are ruled out.

A reduced-form approximation that captures both the UIP carry channel and the commodity terms-of-trade channel is:

$$\ln S_t = \alpha_0 - \alpha_1(r_{US,t} - r_{CA,t}) + \alpha_2 \ln G_t + \alpha_3 \ln P_{oil,t} + \varepsilon_{S,t} \quad (1)$$

where:

1. $r_{US,t} = i_{US,t} - \pi^e_{US,t}$ and $r_{CA,t} = i_{CA,t} - \pi^e_{CA,t}$ are ex-ante real interest rates. The use of real rather than nominal rates reflects the fact that long-run capital flows respond to real returns, and it also removes the direct inflation effect that would otherwise enter via PPP.
2. G_t is the USD price of gold. For Canada, a rise in G_t means higher export revenue from gold, so the CAD appreciates (S_t increases).
3. $P_{oil,t}$ is the USD price of oil. This is included because oil dominates Canadian trade.

2.1.3. Parameter signs and economic mechanisms

Gold and oil prices enter the exchange rate equation because they are key determinants of Canada's terms of trade and external balance.

Gold is a significant Canadian export, accounting for approximately 5% of mine production and about 3% of total merchandise exports. A rise in the USD price of gold directly improves Canada's terms of trade, increasing export revenues and attracting capital inflows.

Oil is Canada's principal export commodity, accounting for over 20% of total exports and approximately 10% of GDP. Consequently, changes in oil prices exert a first-order effect on Canada's terms of trade and trade balance.

Accordingly:

1. $\alpha_1 > 0$ – a higher US real rate (relative to Canada) attracts capital flows into USD assets, leading to an appreciation of the US dollar and a depreciation of the Canadian dollar.
2. $\alpha_2 > 0$ – as gold is an important Canadian export, an increase in the USD price of gold directly improves the CAD's terms of trade. Estimates reported in the literature (Cashin et al., 2002; Chen & Rogoff, 2003) suggest an elasticity in the range of 0.05–0.10.
3. $\alpha_3 > 0$ – oil has a larger coefficient than gold due to Canada's much higher export share. Indicative estimates (Amano & van Norden, 1995) suggest an elasticity in the range of 0.10–0.20.

2.1.4. Inclusion of output gaps and growth

While not explicitly present in Equation (1), the output gaps $g_{US,\,t}$ and $g_{CA,\,t}$ influence the exchange rate through the following channels:

1. **Monetary policy expectations**: Central banks systematically respond to output gaps. A positive US output gap may signal future rate hikes, affecting $r_{US,t}$ with a lag.
2. **Risk appetite and capital flows**: Stronger US growth often boosts global risk appetite, which tends to favour commodity currencies like the CAD.
3. **Commodity demand**: Stronger global growth increases demand for commodities, including gold and oil, which then feeds through to the exchange rate via the commodity price channels.

In an extended version, one could include the output gap differential directly:

$$S_t = \alpha_0 - \alpha_1(r_{US,\,t} - r_{CA,\,t}) + \alpha_2 G_t + \alpha_3 P_{oil,\,t} + \alpha_4(g_{US,\,t} - g_{CA,\,t}) + \varepsilon_{S,\,t}$$

with $\alpha_4 > 0$ if stronger Canadian growth relative to the US appreciates the CAD. However, to maintain parsimony and avoid multicollinearity, we keep the simpler specification.

2.1.5. The error term $\varepsilon_{S,\,t}$

The term $\varepsilon_{S,\,t}$ captures short-run deviations from the long-run equilibrium, including central bank intervention, liquidity shocks, temporary mispricing and noise trading.

2.1.6. Relationship to other equations

Equation (1) constitutes the first block of the simultaneous system. The spot price of gold G_t appears on the right-hand side. In turn, the exchange rate S_t influences the spot price of gold G_t through the USD index), because gold is denominated in USD. This interaction gives rise to a feedback mechanism, which is analysed in Section 3.

2.2. Equation 2: spot price of gold

Gold is a unique asset: it pays no coupon or dividend, incurs storage costs, but serves as a store of value, an inflation hedge and a safe haven. Its price is determined globally in US dollars. The equilibrium condition balances the opportunity cost of holding gold against its perceived benefits as a monetary and speculative asset.

2.2.1. Theoretical foundation

Consider an investor choosing between holding gold and investing in a US Treasury bill that offers a real return $r_{US,t}$. In equilibrium, the expected real return on gold must equal the real interest rate, after accounting for the convenience yield and the risk premium. Solving the model forward under the assumption of rational expectations yields:

$$G_t = \beta_0 - \beta_1 r_{US,t} + \beta_2 \pi^e_{US,t} + \beta_3 \Phi_t + \beta_4 Risk_t + \varepsilon_{G,t} \tag{2}$$

with $\beta_1 > 0$, $\beta_2 > 0$, $\beta_3 > 0$. The term $\pi^e_{US,t}$ captures the inflation-hedging demand.

Assumption 2. The forward solution for gold prices is derived under the assumptions that: (i) gold is an asset that pays no dividend or coupon; (ii) the risk-free real interest rate is the relevant discount rate; (iii) investors form rational expectations with complete information; and (iv) the price of gold does not exhibit explosive dynamics, ensuring that the present value relationship is well-defined.

2.2.2. The role of US real rates and inflation expectations

The coefficient β_1 measures the sensitivity of the gold price to the US real interest rate. A one-percentage-point increase in r_{US} is typically associated with a 5–10% decline in the price of gold in the short run. Inflation expectations $\pi^e_{US,t}$ enter the equation separately from $r_{US,t}$. Empirically, β_2 is often smaller than β_1 because inflation expectations are already partially reflected in r_{US} through the Fisher equation.

2.2.3. Financialisation (Φ_t)

The financialisation index Φ_t is constructed as the first principal component of three normalised series: (i) total gold ETF holdings (in tonnes), (ii) COMEX gold futures open interest (in contracts), and (iii) the CFTC net non-commercial long position (normalised by open interest). This approach follows the methodology of Cheng and Xiong (2014) and Singleton (2014), capturing both the level of institutional participation and the speculative intensity in gold markets. Each component is standardised to zero mean and unit variance before aggregation. The coefficient $\beta_3 > 0$ captures the effect of non-fundamental demand.

2.2.4. Risk sentiment (Risk_t)

Gold is widely regarded as a safe-haven asset. During typical risk-off episodes, gold appreciates as investors seek refuge ($\beta_4 > 0$). However, during extreme liquidity crises, investors sell gold alongside other assets to raise cash, leading to a temporary decline in its price. Consequently, the sign of β_4 is regime-dependent.

2.2.5. The role of economic growth

Economic growth affects gold prices through two channels. First, **commodity demand**: stronger global growth (particularly from the US and emerging economies) increases industrial demand for gold (jewellery, electronics). Second, **monetary policy expectations**: stronger growth may signal future rate hikes, which could lower gold prices through the real rate channel, but this effect is already captured through $r_{US,t}$. The net effect is ambiguous and likely regime-dependent.

2.2.6. Why Canadian real rates do not appear directly

Gold is priced in US dollars and traded globally. Consequently,the opportunity cost of holding gold is determined primarily by the US real interest rate. Canadian real interest rates $r_{CA,t}$ affect gold only indirectly through the exchange rate channel and the strong correlation between US and Canadian real interest rates. Including both variables directly in the gold price equation would introduce multicollinearity without providing additional theoretical insight.

2.2.7. Error term $\varepsilon_{G,t}$

The error term captures temporary supply shocks, high-frequency noise and measurement errors. While some studies find that gold prices and real interest rates are not cointegrated over long samples (Erb & Harvey, 2013), others find evidence of a long-run relationship when allowing for structural breaks. Our model accommodates instabilities through the financialisation and risk terms.

2.2.8. Incorporating exchange rate feedback

Equation (2), in its baseline form, does not include the exchange rate directly. However, as gold is denominated in USD, changes in the USD/CAD exchange rate affect the effective USD price of gold for non-US investors. To capture this mechanism and close the model, Equation (2) is augmented with an explicit exchange rate term:

$$G_t = \beta_0 - \beta_1 r_{US,t} + \beta_2 \pi^e_{US,t} + \beta_3 \Phi_t + \beta_4 Risk_t - \gamma \ln(\text{USD Index}_t) + \varepsilon_{G,t}$$

Since

$$\ln(\text{USD Index}_t) = -\ln S_t + C$$

where C is a constant, Equation (3) can be rewritten as

$$G_t = \beta_0 - \beta_1 r_{US,t} + \beta_2 \pi^e_{US,t} + \beta_3 \Phi_t + \beta_4 \text{Risk}_t + \gamma \ln S_t + \varepsilon_{G,t} \qquad (3)$$

with $\gamma > 0$. This augmented specification is used for the feedback analysis presented in Section 3. For the baseline model (without exchange rate feedback), we set $\gamma = 0$.

2.3. Equation 3: gold futures and financialisation

Gold futures are exchange-traded contracts that obligate the seller to deliver physical gold at a specified future date. The relationship between the futures price F_t and the spot price G_t is governed by arbitrage, but in the presence of financialisation and limits to arbitrage, deviations can persist.

2.3.1. The no-arbitrage cost of carry model

The no-arbitrage condition in the presence of a convenience yield is:

$$F_t^{NA} = G_t e^{(r_{US,t} + c - y)T}$$

If $F_t > G_t e^{(r_{US,t} + c - y)T}$, arbitrageurs can earn a risk-free profit by selling futures contracts, purchasing physical gold, storing it and delivering it at contract maturity. Conversely, short selling physical gold is often difficult, so the lower bound may not be enforceable.

2.3.2. Components of the cost of carry

The cost of carry comprises three principal components:

1. **Real interest rate** ($r_{US,\,t}$). A higher r_{US} real interest rate increases the opportunity cost of holding physical gold and, consequently, raises the futures price relative to the spot price, thereby widening the contango.
2. **Storage and insurance cost** (c). For gold, storage and insurance costs are generally low, typically ranging from 0.1% to 0.2% per annum.
3. **Convenience yield** (y). The convenience yield represents the non-monetary benefits of holding physical gold. When physical gold becomes relatively scarce or highly valued for immediate delivery, the convenience yield increases, reducing the futures price relative to the spot price and potentially leading to backwardation.

2.3.3. Limits to arbitrage and the financialisation premium

Under the assumptions of perfectly competitive markets, the no-arbitrage condition holds exactly. In practice, however, gold markets are subject to a range of frictions including constraints on short selling physical gold, regulatory and margin requirements, as well as financialisation-driven investment demand. We capture these frictions by adding a premium $\psi\Phi_t^{spec}$ to the no-arbitrage price, where Φ_t^{spec} is the speculative subcomponent of the financialisation index.

$$F_t = G_t e^{(r_{US,\,t} + c - y)T} + \psi\Phi_t^{spec} + \varepsilon_{F,\,t} \tag{4}$$

with $\psi > 0$. The error term $\varepsilon_{F,\,t}$ captures transient mispricings.

2.3.4. Interpreting the futures curve

Equation (3) implies that the observed basis decomposes into a fundamental part (cost of carry) and a speculative part. A negative basis (backwardation) can arise either from a high convenience yield y or from negative speculative pressure. A positive basis (contango) can be due to low y or high speculative length Φ_t^{spec}.

2.3.5. The role of convenience yield dynamics

Convenience yield y is not directly observable. For empirical work, we treat y as a latent variable or proxy it with measures of physical gold demand. Alternatively, we can rewrite Equation (3) as:

$$F_t = G_t e^{(r_{us,t} + c)T} - \delta_t + \psi \Phi_t^{spec} + \varepsilon_{F,t}$$

where $\delta_t = G_t e^{(r_{us,t} + c)T}(1 - e^{-y\mathrm{T}})$ is the convenience yield in dollar terms.

2.3.6. Connection to USD/CAD and gold spot

Equation (3) introduces additional identifying restrictions. In particular, the coefficient on $r_{US,t}$ inside the exponential must be consistent with the real interest rate used in the gold spot price equation. Furthermore, the financialisation premium Φ_t^{spec} may be correlated with the overall Φ_t in the gold spot price equation. If Φ_t includes speculative futures positions, then β_3 and ψ are jointly identified by the fact that Φ_t affects G_t directly but also affects F_t through the premium.

3. Theoretical analysis and model properties

Having specified the three structural equations, we now examine the properties of the complete system. We first assemble the equations into a simultaneous framework, analyse the cross-equation feedbacks, derive the reduced-form solution, and discuss the stability conditions and comparative statics. This analysis reveals the amplification mechanisms and provides testable predictions for empirical validation.

3.1. Simultaneous system and cross-equation feedbacks

Having specified the three structural equations, we can now assemble them into a complete simultaneous system.

3.1.1. The complete model

The full theoretical model consists of the following three equations:

$$S_t = \alpha_0 - \alpha_1(r_{US,t} - r_{CA,t}) + \alpha_2 G_t + \alpha_3 P_{oil,t} + \varepsilon_{S,t} \qquad (1)$$

$$G_t = \beta_0 - \beta_1 r_{US,t} + \beta_2 \pi^e_{US,t} + \beta_3 \Phi_t + \beta_4 \mathrm{Risk}_t + \gamma \ln S_t + \varepsilon_{G,t} \qquad (2')$$

$$F_t = G_t e^{(r_{US,t} + c - y)T} + \psi \Phi_t^{spec} + \varepsilon_{F,t} \qquad (3)$$

Equations (1) and (2’) are simultaneous: G_t appears in Equation (1) for S_t, and S_t appears in Equation (2’) for G_t. Equation (3) is recursive given G_t and $r_{US,t}$.

3.1.2. Matrix representation and reduced form

For small changes, we linearise the exponential term in Equation (3):

$$F_t \approx G_t\left[1+(r_{US,t}+c-y)\,T\right]+\psi\Phi_t^{spec}+\varepsilon_{F,t}$$

Define $\phi = (c - y)T$ as a constant. The resulting linearised system can be written in matrix form as:

$$\begin{pmatrix} 1 & -\alpha_2 & 0 \\ -\gamma & 1 & 0 \\ 0 & -\left[1+(r_{US}+c-y)\,T\right] & 1 \end{pmatrix}\begin{pmatrix} S_t \\ G_t \\ F_t \end{pmatrix}=$$

$$=\begin{pmatrix} \alpha_0-\alpha_1(r_{US,t}-r_{CA,t})+\alpha_3 P_{oil,t}+\varepsilon_{S,t} \\ \beta_0-\beta_1 r_{US,t}+\beta_2\pi^e_{US,t}+\beta_3\Phi_t+\beta_4\mathrm{Risk}_t+\varepsilon_{G,t} \\ \psi\Phi_t^{spec}+\varepsilon_{F,t} \end{pmatrix}$$

The reduced-form solution for S_t and G_t is then obtained as:

$$\begin{pmatrix} S_t \\ G_t \end{pmatrix}=\frac{1}{1-\alpha_2\gamma}\begin{pmatrix} 1 & \alpha_2 \\ \gamma & 1 \end{pmatrix}\cdot(\text{exogenous terms})$$

The multiplier $1/(1 - \alpha_2\gamma)$ plays a crucial role in determining the strength and stability of the feedback mechanism. In a dynamic setting, the stability condition ensures that successive feedback effects diminish over time, preventing explosive dynamics:

$$G_{t+1}=\gamma S_t,\;\; S_{t+1}=\alpha_2 G_t$$

which has the characteristic equation $\lambda^2 = \alpha_2\gamma$. The system is stable if $|\lambda| < 1$, i.e., $\alpha_2\gamma < 1$. Empirical evidence suggests that $\alpha_2 \approx 0.05–0.10$ and $\gamma \approx 0.8–1.0$, so $\alpha_2\gamma \approx 0.04–0.10$, well below 1.

3.1.3. Detailed analysis of feedback loops

Loop 1: Gold $\rightarrow$ CAD

A positive shock to G_t directly increases S_t by α_2. This is the direct commodity-currency channel.

Loop 2: CAD → gold (via USD)

The appreciation of CAD (rise in S_t) means the USD weakens. Since gold is priced in USD, a weaker USD makes gold cheaper for non-US investors, increasing demand and pushing G_t up. The elasticity γ is approximately 0.8–1.0.

The combined effect is:

$$\frac{dG_t}{d\varepsilon_{G,t}} = \frac{1}{1-\alpha_2\gamma} \cdot (\text{direct effect})$$

Loop 3: US real rates

A rise in $r_{US,t}$ has two opposing effects:

1. Directly lowers G_t (via β_1).
2. Widens the US-Canada real interest differential, causing S_t to fall (CAD depreciates, USD appreciates). A stronger USD further reduces G_t.

The total effect (accounting for the feedback loop but abstracting from the amplification multiplier for clarity) is:

$$\frac{dG_t}{dr_{US}} = -\beta_1 - \alpha_1\gamma$$

The full comparative-static effect, which includes the amplification mechanism, is presented in Appendix A.2.

Loop 4: financialization

A surge in Φ_t directly increases G_t by β_3. Higher G_t then appreciates CAD, which weakens the USD and provides a second boost to G_t. The total effect is:

$$\frac{dG_t}{d\Phi_t} = \frac{\beta_3}{1-\alpha_2\gamma}$$

Loop 5: oil price

A rise in oil directly appreciates CAD ($\alpha_3 > 0$). The stronger CAD then affects gold through the exchange rate channel: a stronger CAD means a weaker USD, which raises the USD price of gold. Thus:

$$\frac{dG_t}{dP_{oil}} = \frac{\alpha_3\gamma}{1-\alpha_2\gamma} > 0$$

Risk aversion

The risk aversion effect discussed here represents an economic interpretation of the model's qualitative predictions, but is not formally derived from the model equations. To formally incorporate safe-haven dynamics, one would need to extend the model with endogenous risk premiums, which we leave for future research.

3.1.4. Stability and long-run equilibrium

The stability condition $\alpha_2\gamma < 1$ is almost certainly satisfied. The long-run equilibrium is obtained by setting the error terms to zero and assuming that all exogenous variables are at their steady-state values:

$$S^{LR} = \alpha_0 - \alpha_1\left(r_{US}^{LR} - r_{CA}^{LR}\right) + \alpha_2 G^{LR} + \alpha_3 P_{oil}^{LR}$$
$$G^{LR} = \beta_0 - \beta_1 r_{US}^{LR} + \beta_2 \pi_{US}^{e,\,LR} + \beta_3 \Phi^{LR} + \beta_4 \mathrm{Risk}^{LR} + \gamma S^{LR}$$
$$F^{LR} = G^{LR} e^{\left(r_{US}^{LR} + c - y\right)T} + \psi \Phi^{spec,\,LR}$$

These equations describe the long-run levels of the endogenous variables conditional on the steady-state values of the exogenous variables. They do not represent a dynamic solution; rather, they provide the equilibrium around which the variables fluctuate in response to shocks.

3.1.5. Summary of cross-equation feedbacks

Table 1 summarises the direction of each key feedback.

Table 1. Direction of cross-equation feedbacks

Shock	Effect on G_t	Effect on S_t
G_t ↑ (direct)	–	↑ (CAD appreciates)
S_t ↑ (CAD appreciates)	↑ (via USD)	–
r_{US} ↑	↓ (direct + via USD)	↓ (CAD depreciates)
Φ_t ↑	↑ (amplified by feedback)	↑ (amplified)
P_{oil} ↑	↑ (via CAD appreciation)	↑

Source: own elaboration based on the theoretical model presented in Section 3.

3.2. Theoretical predictions

Table 2 summarises the sign predictions for selected shocks.

Table 2. Sign predictions for selected shocks

Shock	USD/CAD	ΔG_t	$\Delta(F_t - G_t e^{(\cdot)T})$
$r_{US}\uparrow$	CAD ↓	↓↓	contango widens
$r_{CA}\uparrow$	CAD ↑	≈ 0	≈ 0
US growth ↑	ambiguous	ambiguous	backwardation if physical demand rises
$\Phi_t\uparrow$	CAD ↑	↑↑	futures premium ↑
risk ↑	CAD ↓	↑	backwardation if hoarding

Source: own elaboration based on the theoretical model presented in Section 3.

4. Preliminary empirical validation

To ground the theoretical model in data, we conducted a preliminary empirical validation using monthly data from January 2000 to December 2024.

4.1. Data description

The following variables are used:

1. **USD/CAD exchange rate**: Bank of Canada monthly average.
2. **Gold spot price**: London Bullion Market Association (LBMA) PM fix (USD/oz).
3. **Gold futures price**: COMEX GC1 front-month contract, converted to constant 3-month maturity via interpolation.
4. **US real interest rate**: 10-year TIPS yield.
5. **Canadian real interest rate**: 10-year Government of Canada bond yield minus 10-year Consensus Economics inflation forecast.
6. **US inflation expectations**: 5-year breakeven inflation rate.
7. **Financialisation index**: First principal component of (i) GLD ETF holdings (tonnes), (ii) COMEX open interest, (iii) CFTC net non-commercial long position.

8. **Risk**: VIX index (monthly average).
9. **Oil price**: WTI crude oil price (USD/barrel).

All variables are transformed to logarithms (except interest rates, which are in percentage points).

4.2. Cointegration analysis

We first test for cointegration using the Johansen procedure. The ADF tests indicate that s_t, g_t, f_t, and oil_t are $I(1)$, while $r_{US,t}$, $r_{CA,t}$, $\pi^e_{US,t}$, Φ_t, and Risk_t are $I(0)$. The Johansen trace test (with 4 lags) indicates 1 cointegrating relationships at the 5% significance level (Table 3).

Table 3. Johansen cointegration test results

Hypothesised No of CEs	Trace statistic	5% critical value	p-value	Max-Eigen statistic
None	78.45	63.88	0.008	42.13
At most 1	36.32	42.92	0.064	22.87
At most 2	13.45	25.87	0.251	11.56
At most 3	1.89	12.52	0.521	1.89

Source: own calculations based on data described in Section 4.1.

The normalised cointegrating vectors are:

Vector 1: $s_t - 0.45r_{US,t} + 0.38r_{CA,t} - 0.08g_t - 0.15oil_t + 0.06\Phi_t = \text{stationary}$

Vector 2: $g_t + 0.52r_{US,t} — 0.31\%\pi^e_{US,t} - 0.28\Phi_t - 0.12\mathrm{Risk}_t + 0.05s_t = \text{stationary}$

These vectors are consistent with the theoretical predictions: the signs match those in Equations (1) and (2’), and the magnitudes are in the expected ranges.

4.3. Structural VAR analysis

We estimate a structural VAR with sign restrictions to identify the shocks. The identification scheme uses the sign restrictions presented in Table 4.

Table 4. Sign restrictions for SVAR identification

Shock	S_t	G_t	F_t	$r_{US,t}$
US real rate shock	–	–	+	+
Financialisation shock	+	+	+	0
Risk shock	–	+	0	0
Oil shock	+	+	0	0

Source: own elaboration based on the theoretical predictions of the model in Section 3.2.

The impulse response functions show:

1. A positive US real rate shock lowers both the CAD and gold prices, with gold falling by approximately 6% after 3 months.
2. A positive financialisation shock raises both gold and the CAD, with gold rising by approximately 8% after 3 months.
3. A positive risk shock raises gold but lowers the CAD (consistent with safe-haven flows into USD).

Table 5. SVAR impulse response point estimates and 90% confidence intervals

Shock	Variable	1-month	3-month	6-month	12-month
US real rate	S_t	–0.012	–0.034	–0.021	–0.008
		[–0.019, –0.005]	[–0.051, –0.017]	[–0.038, –0.004]	[–0.019, 0.003]
US real rate	G_t	–0.025	–0.058	–0.041	–0.018
		[–0.041, –0.009]	[–0.087, –0.029]	[–0.071, –0.011]	[–0.042, 0.006]
Financialisation	S_t	0.018	0.045	0.032	0.015
		[0.009, 0.027]	[0.028, 0.062]	[0.016, 0.048]	[0.004, 0.026]
Financialisation	G_t	0.032	0.078	0.051	0.022
		[0.017, 0.047]	[0.052, 0.104]	[0.030, 0.072]	[0.009, 0.035]
Risk	S_t	–0.015	–0.028	–0.012	–0.005
		[–0.024, –0.006]	[–0.045, –0.011]	[–0.028, 0.004]	[–0.018, 0.008]
Risk	G_t	0.022	0.041	0.018	0.009
		[0.011, 0.033]	[0.019, 0.063]	[–0.002, 0.038]	[–0.007, 0.025]

Source: author's own calculations based on data from sources listed in Table B1.

4.4. Error correction model

The error correction model (ECM) estimates are:

$$\Delta s_t = -0.12(s_{t-1} - 0.45 r_{US,\,t-1} + 0.38 r_{CA,\,t-1} - 0.08 g_{t-1} - 0.15 oil_{t-1} - 0.06\Phi_{t-1}) + \\ + 0.05\,\Delta g_t + 0.12\Delta oil_t + \varepsilon_{S,\,t}$$

$$\Delta g_t = -0.08(g_{t-1} + 0.52 r_{US,\,t-1} - 0.28\Phi_{t-1} - 0.12\mathrm{Risk}_{t-1} - 0.05 s_{t-1}) + 0.15\Delta r_{US,\,t} + \\ + 0.22\Delta\Phi_t + \varepsilon_{G,\,t}$$

The coefficients are all statistically significant at the 5% level and have the expected signs. The error correction speeds (–0.12 and –0.08) indicate moderate adjustment toward equilibrium.

Table 6. Error correction model coefficient estimates

Equation	Variable	Coefficient	Std. error	*t*-statistic
Δs_t	error correction term	–0.124	0.038	–3.26
	Δg_t	0.052	0.021	2.48
	Δoil_t	0.118	0.034	3.47
Δg_t	error correction term	–0.082	0.031	–2.65
	$\Delta r_{US,\,t}$	–0.152	0.045	–3.38
	$\Delta\Phi_t$	0.218	0.057	3.82

Source: author's own calculations based on data from sources listed in Table B1.

4.5. Identification and estimation issues

The simultaneous feedback mechanisms give rise to several estimation challenges:

1. **Simultaneity bias**. OLS on the CAD equation will produce biased estimates of instrumental variables; consequently, system methods (3SLS, FIML) are required.
2. **Identification via sign restrictions**. The sign restrictions derived from the feedback loops can be used to identify the structural shocks.

3. **Weak instrument concern**. Financialisation proxies may be weak instruments if they do not have sufficient exogenous variation.
4. **Cointegration vs differencing**. Given the mixed integration properties of the variables, a cointegration framework (VECM) is appropriate for the long-run relationships.
5. **Time-varying parameters**. Coefficients may change over time (ZLB, financialisation regimes), suggesting the use of time-varying parameter or regime-switching models.

4.6. Robustness checks

To assess the robustness of the empirical results, several additional specifications are considered:

1. **Alternative measures of the real interest rate.** Replacing the 10-year TIPS yield with the 3-month TIPS yield yields qualitatively similar results, although the estimated coefficients are slightly smaller in magnitude.
2. **Alternative financialisation index construction**: Replacing the composite financialisation index with either ETF holdings alone or CFTC speculative positions alone, instead of the first principal component, produces qualitatively similar results, with only minor differences in the estimated coefficients.
3. **Stability tests**. Chow and CUSUM tests indicate that the cointegrating relationships remain stable over the sample period, with the exception of the 2008–2009 global financial crisis.

Table 7. Results of the robustness checks

Variant	Key coefficient	Sign consistent?
Baseline (10-year TIPS)	$\alpha_2 = 0.082$	yes
Alternative (3-month TIPS)	$\alpha_2 = 0.065$	yes
ETF-only financialisation	$\beta_3 = 0.185$	yes
COT-only financialisation	$\beta_3 = 0.152$	yes
Pre-2008 subsample	$\alpha_2 = 0.071$	yes
Post-2008 subsample	$\alpha_2 = 0.094$	yes

Source: author's own calculations based on data from sources listed in Table B1.

5. Discussion and implications

The theoretical framework and preliminary empirical validation yield several important implications. As with any empirical analysis, these implications should be interpreted with caution. The empirical results presented in Section 4 are based on a specific sample period (2000–2024) and may not generalise to other periods or contexts. Furthermore, the identification of structural parameters relies on the validity of the sign restrictions and the underlying theoretical assumptions. Future research with more comprehensive data and alternative identification strategies may refine or modify these implications.

5.1. Policy implications

The analysis highlights the strong interdependence between exchange rates, commodity prices and financial markets. Although the policy implications should be interpreted with caution and remain subject to further empirical validation, several conclusions emerge from the theoretical framework.

1. **Monetary policy transmission**. The amplification of US real interest rate shocks through the CAD-gold feedback loop implies that monetary policy in the US may have larger effects on Canadian variables than standard models would suggest. This finding, if confirmed in future research, would have important implications for the Bank of Canada's policy reaction function.
2. **Financial stability**. The financialisation channel suggests that speculative flows can cause significant deviations from fundamental values, which may have implications for macroprudential policy. Our preliminary evidence indicates that regulatory attention to commodity futures markets may be warranted.
3. **Exchange rate management**. The feedback loop between CAD and gold implies that the Bank of Canada's exchange rate policy may have unintended effects on gold prices and, through that channel, back onto the CAD. This suggests that policymakers should consider the global commodity price implications of their actions.

5.2. Investment implications

The proposed framework also has potential implications for market participants.

1. **Cross-market hedging**. The positive correlation between gold and CAD suggests that gold can serve as a partial hedge for CAD exposure. However, this relationship may be regime-dependent and should be monitored over time.
2. **Carry trade strategies**: The real interest differential, amplified through the commodity channel, may provide better signals for carry trades than nominal differentials. Our empirical results suggest that incorporating gold prices could improve carry trade signal quality.
3. **Futures positioning**. The speculative premium in futures prices may provide signals for spot market movements, as it reflects the financialisation channel. Investors should be aware that futures positioning data may contain information about future spot price movements.

5.3. Limitations and future research

Several limitations of the current analysis suggest avenues for future research.

1. **Nonlinearities**. The relationship may be nonlinear, especially at the zero lower bound or during financial crises. Regime-switching models could address this.
2. **Endogeneity of financialisation**. The financialisation index may itself be endogenous to gold price movements. Future work could use exogenous instruments (e.g. regulatory changes, ETF creation/redemption).
3. **Other commodities**. The framework could be extended to include other Canadian exports (e.g. lumber, natural gas).
4. **Microstructure effects**. High-frequency dynamics and market microstructure effects could be incorporated into a more detailed model.
5. **Cross-country validation**. Extending the framework to other commodity currencies (AUD, NZD, NOK) would provide additional evidence on the generalisability of our findings.

Conclusions

This paper develops a coherent theoretical framework that links the USD/CAD exchange rate, the spot price of gold, and gold futures prices using real interest rates, economic growth and financialisation in both the US and Canada. The proposed model highlights the dual role of gold as both a commodity (affecting CAD through the terms of trade) and a financial asset (driven by US real interest rates and speculative flows). The futures pricing equation extends the traditional no-arbitrage condition by incorporating a financialisation premium.

The principal findings may be summarised as follows:

1. The feedback between CAD and gold creates an amplification mechanism $(1/(1 - \alpha_2\gamma))$ that can magnify shocks.
2. US real interest rates are a primary driver of both gold and CAD, with the indirect effects reinforcing the direct ones.
3. Financialisation shocks can decouple gold from fundamentals and, through the exchange rate channel, affect the CAD.
4. The preliminary empirical validation confirms the model's core predictions and provides coefficient estimates consistent with the underlying theoretical priors.

The system yields testable cross-equation restrictions and provides a foundation for empirical analysis of the interactions between exchange rates, commodity prices and futures markets. The framework can also be extended to include other commodity currencies, nonlinear dynamics and high-frequency microstructure effects.

Appendix

A. Mathematical properties of the system

A.1. Existence and uniqueness

The system matrix in (A.3) is invertible if its determinant is non-zero:

$$\det(A) = 1 - \alpha_2\gamma$$

The solution therefore exists and is unique if and only if $1 - \alpha_2\gamma \neq 0$. Empirical estimates suggest that $0 < \alpha_2\gamma < 1$, with $\alpha_2 \approx 0.05–0.10$ and $\gamma \approx 0.8–1.0$; consequently, the system matrix is invertible, and a unique solution exists for any admissible set of exogenous variables.

A.2. Comparative statics

The full comparative-static effects for the extended model are:

$$\frac{dS}{dr_{US}} = \frac{-\alpha_1 + \alpha_2(-\beta_1)}{1-\alpha_2\gamma} < 0$$
$$\frac{dG}{dr_{US}} = \frac{-\beta_1 + \gamma(-\alpha_1)}{1-\alpha_2\gamma} < 0$$
$$\frac{dS}{d\Phi} = \frac{\alpha_2\beta_3}{1-\alpha_2\gamma} > 0$$

$$\frac{dG}{d\Phi} = \frac{\beta_3}{1-\alpha_2\gamma} > 0$$
$$\frac{dS}{dP_{oil}} = \frac{\alpha_3}{1-\alpha_2\gamma} > 0$$
$$\frac{dG}{dP_{oil}} = \frac{\gamma\alpha_3}{1-\alpha_2\gamma} > 0$$

These expressions account for both the feedback loop and the amplification mechanism inherent in the simultaneous system. The denominator $1 - \alpha_2\gamma > 0$ when $\alpha_2\gamma < 1$, which amplifies the effects.

A.3. Local stability

To analyse stability, we consider an auxiliary dynamic system that describes a hypothetical adjustment process toward the equilibrium of the static model. This system is not a structural equation or a direct transformation of the original economic model; rather, it is a mathematical device used to examine whether small deviations from equilibrium would converge back to it under a plausible adjustment mechanism.

Consider the auxiliary dynamic system:

$$S_{t+1} = \alpha_2 G_t$$
$$G_{t+1} = \gamma S_t$$

The system matrix (transition matrix) is:

$$A = \begin{pmatrix} 0 & \alpha_2 \\ \gamma & 0 \end{pmatrix}$$

Proposition 1 (stability of the feedback system)**.** Consider the auxiliary dynamic system $S_{t+1} = \alpha_2 G_t$ and $G_{t+1} = \gamma S_t$. The equilibrium $(S^*,\ G^*)$ is locally stable if and only if $\alpha_2\gamma < 1$.

Proof. The system matrix is $A = \begin{pmatrix} 0 & \alpha_2 \\ \gamma & 0 \end{pmatrix}$. The eigenvalues are the solutions to $|A - \lambda I| = 0$, yielding:

$$\det\begin{pmatrix} -\lambda & \alpha_2 \\ \gamma & -\lambda \end{pmatrix} = \lambda^2 - \alpha_2\gamma = 0$$

Thus, $\lambda_{1,2} = \pm\sqrt{\alpha_2\gamma}$. The equilibrium is stable if $|\lambda_{1,2}| < 1$, which is equivalent to $\alpha_2\gamma < 1$. QED.

Since the system is linear, the system matrix is constant and the eigenvalues directly characterise the stability of the equilibrium. Empirically, $\alpha_2\gamma < 1$, so the equilibrium is stable.

B. Data appendix

Table B1 presents detailed variable definitions and sources.

Table B1. Variable definitions and sources

Variable	Definition	Source
USD/CAD	Monthly average exchange rate	Bank of Canada
Gold spot	LBMA PM fix (USD/oz)	Refinitiv
Gold futures	COMEX GC1 constant 3-month	Refinitiv
US real rate	10-year TIPS yield	FRED
CA real rate	1 0-year bond yield minus inflation forecast	BoC, Consensus Economics
US inflation exp.	5-year breakeven inflation	FRED
Financialisation	First PC of ETF holdings, OI, COT	Bloomberg, CFTC
Risk	VIX monthly average	CBOE
Oil price	WTI (USD/barrel)	EIA

Source: own study.

C. Implementation notes

For researchers wishing to replicate our empirical analysis presented in this paper, the following implementation guidelines are recommended:

1. All variables should be transformed to stationarity as appropriate: log levels for S_t, G_t, F_t, and $P_{oil,\,t}$; levels for interest rates and the financialisation index.
2. The Johansen cointegration test should include a constant and a linear trend in the long-run relationships.
3. The SVAR sign restrictions should be implemented using the algorithm of Johansen (1988) or Uhlig (2005).
4. The error correction model should be estimated using maximum likelihood with appropriate lag selection (AIC or BIC).
5. The matrix representation and comparative statics are based on a log-linear approximation around the steady state, where $\gamma \ln S_t$ is approximated as $\gamma \tilde{s}_t$ (the log deviation of S_t). This is standard for local dynamics and valid for small deviations.

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