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A turnpike theorem for a non-stationary Gale economy with limit technology. A particular case¹

*Emil Panek*²

Abstract: This paper is a continuation of the work by Panek [2013a], [2013b] and Panek and Runka [2011], with direct reference to [2013b]. The difference between this paper and paper [2013b] reveals when applying a different optimality criterion with the help of which growth processes are assessed. The optimality criterion in [2013b] is the value of production in the last period of a fixed planning horizon $T = \{0, 1, \dots, t_1\}$ measured in von Neumann equilibrium prices. In this paper we take the total value of output (also measured in von Neumann prices) produced in the whole horizon T as a growth criterion. The aim of the paper is to prove that a change of growth criterion does not deprive the economy of its turnpike properties. It was pointed out that a Gale economy with a changeable technology convergent to the limit technology also preserves similar properties if we accept the social utility function (with standard properties) as a growth criterion.

The paper consists of three sections. In Section 1 we present a non-stationary Gale-type model with a limit technology. Section 2 contains proof of the so-called ‘weak’ turnpike theorem, whilst in Section 3 we prove a ‘strong’ version of the turnpike theorem for the economy under investigation. The paper closes with final comments.

Keywords: non-stationary Gale economy, limit technology, von Neumann equilibrium, turnpike.

JEL codes: C10.

1. A model of non-stationary Gale economy with limit technology

The model that we shall consider has been presented in detail in Panek [2013b]. Here we give a brief description of the facts that are necessary for understanding of what follows.

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The economy we consider works in (discrete) time periods $t = 0, 1, 2, \dots$ producing and/or utilizing n goods. By $x(t) = (x_1(t), \dots, x_n(t)) \geq 0$ we denote an input vector in period t , by $y(t) = (y_1(t), \dots, y_n(t)) \geq 0$ we denote an output vector in period t . A pair $(x(t), y(t))$ is said to describe a (technologically) feasible production process in period t . The set of all feasible production processes is denoted by $Z(t)$ and it is called a production space in period t . Production spaces are defined for all integers $t \geq 0$ and are assumed to meet the following conditions, see Panek [2003; Ch. 5], Panek [2013b; Sect. 1]:

(G1) $\forall (x, y) \in Z(t) \forall \lambda \geq 0 (\lambda(x, y) \in Z(t))$.

(G2) $\forall (x^i, y^i), i = 1, 2 ((x^1 + x^2, y^1 + y^2) \in Z(t))$.

(G3) $\forall (x, y) \in Z(t) (x = 0 \Rightarrow y = 0)$.

(G4) $\forall (x, y) \in Z(t) (x' \geq x \Rightarrow (x', y) \in Z(t))$.

(G5) $\forall (x, y) \in Z(t) (0 \leq y' \leq y \Rightarrow (x, y') \in Z(t))$.

(G6) $Z(t)$ is a closed subset of R^{2n} .

(G7) $Z(t) \subseteq Z(t+1) \subseteq Z$, where Z is the smallest closed subset of R^{2n} containing all production spaces $Z(t)$ such that the conditions **(G1)–(G6)** are satisfied for Z (with Z in place of $Z(t)$).

Production spaces satisfying the conditions **(G1)–(G6)** are called Gale's production spaces. The inclusion $(x, y) \in Z(t)$ (equivalently $(x(t), y(t)) \in Z(t)$) means that it is possible to produce output y from input x in period t . The set $Z \subset R_+^{2n}$ mentioned in condition **(G7)** is called the limit technology. An economy whose production spaces $Z, Z(t), t = 0, 1, 2, \dots$, fulfill the conditions **(G1)–(G7)** is called a non-stationary Gale economy with limit technology.

□ **Theorem 1.** If production spaces meet conditions **(G1)–(G7)**, then

$$(x, y) \in Z \Leftrightarrow \exists (x(t), y(t)) \in Z(t), t = 0, 1, \dots \left(\lim_t (x(t), y(t)) = (x, y) \right).$$

Proof. See Panek [2013b; Theorem 1]. ■

Let us fix a production process $(x, y) \neq 0$. Then, according to **(G3)**, $x \neq 0$. The number

$$\alpha(x, y) = \max\{\alpha \mid \alpha x \leq y\}$$

is called the technological efficiency rate of the feasible production process (x, y) . The function α is continuous and positively homogenous of degree 0 in $Z \setminus \{0\}$ (and in $Z(t) \setminus \{0\}$), see Panek [2003; Ch. 5, Theorem 5.2]. The number

$$\alpha_{M,t} = \max_{\substack{(x,y) \in Z(t) \\ (x,y) \neq 0}} \alpha(x, y) \tag{1}$$

is called an optimal technological efficiency rate in a non-stationary Gale economy in period t , and the number

$$\alpha_M = \max_{\substack{(x,y) \in Z \\ (x,y) \neq 0}} \alpha(x,y) \quad (2)$$

is called an optimal technological efficiency rate in a non-stationary Gale economy with limit technology.

□ **Theorem 2.** Under assumptions (G1)–(G7):

$$(i) \quad \forall t \exists (\bar{x}(t), \bar{y}(t)) \in Z(t) \left(\alpha(\bar{x}(t), \bar{y}(t)) = \max_{\substack{(x,y) \in Z(t) \\ (x,y) \neq 0}} \alpha(x,y) = \alpha_{M,t} \right),$$

$$\exists (\bar{x}, \bar{y}) \in Z \left(\alpha(\bar{x}, \bar{y}) = \max_{\substack{(x,y) \in Z \\ (x,y) \neq 0}} \alpha(x,y) = \alpha_M \right),$$

$$(2i) \quad \forall t (\alpha_{M,t} \leq \alpha_{M,t+1} \leq \alpha_M).$$

Proof. See Panek [2013b; Theorem 2]. ■

A pair of vectors $(\bar{x}(t), \bar{y}(t))$ satisfying the claim of Theorem 2 is said to be an optimal production process in a non-stationary Gale economy in period t . Similarly a pair (\bar{x}, \bar{y}) is said to be a limit optimal production process in a non-stationary Gale economy. In addition we assume:

$$(G8) \quad \exists (\bar{x}, \bar{y}) \in Z (\alpha(\bar{x}, \bar{y}) = \alpha_M \wedge \bar{y} > 0).$$

An economy satisfying property (G8) is called limit-regular. If a Gale economy meets (G8), then

$$\exists (\bar{x}, \bar{y}) \in Z (\alpha_M \bar{x} = \bar{y} > 0). \quad (3)$$

Whenever we speak of a limit-regular Gale economy we mean an economy for which condition (3) is met. Let us observe that if $(\bar{x}, \bar{y}) \in Z$ is a limit optimal production process satisfying the condition, then³

$$\frac{\bar{x}}{\|\bar{x}\|} = \frac{\alpha_M^{-1} \bar{y}}{\|\alpha_M^{-1} \bar{y}\|} = \frac{\bar{y}}{\|\bar{y}\|} = \bar{s} > 0. \quad (4)$$

³ Here and on: $\|a\| = \sum_i a_i$

The vector \bar{s} characterizes output and input structures in a limit optimal production process (\bar{x}, \bar{y}) . The half-line

$$N = \{\lambda \bar{s} \mid \lambda > 0\}$$

is called a turnpike (or von Neumann's ray) for a non-stationary Gale economy with limit technology.

Let $p = (p_1, \dots, p_n) \geq 0$ denote a vector of goods' prices and let us fix a production process $(x, y) \neq 0$. The number⁴

$$\beta(x, y, p) = \frac{\langle p, y \rangle}{\langle p, x \rangle}$$

(if defined) is called economic efficiency rate of the process (x, y) at prices p .

□ **Theorem 3.** If a Gale economy is limit-regular, then

$$\exists \bar{p} \geq 0 \forall (x, y) \in Z (\langle \bar{p}, y \rangle - \alpha_M \langle \bar{p}, x \rangle \leq 0) \quad (5)$$

and

$$\beta(\bar{x}, \bar{y}, \bar{p}) = \max_{\substack{(x, y) \in Z \\ (x, y) \neq 0}} \beta(x, y, \bar{p}) = \alpha(\bar{x}, \bar{y}, \bar{p}) = \alpha_M.$$

Proof. See Panek [2003; Ch. 5, Theorems 5.3, 5.4]. ■

A process $(\bar{x}, \bar{y}) \in Z$ and prices \bar{p} are said to characterize a non-stationary Gale economy with limit technology in a von Neumann equilibrium. In such an equilibrium the economic efficiency rate and the greatest possible technological efficiency rate are equal. The prices are called von Neumann equilibrium prices (in a non-stationary Gale economy with limit technology).

The next condition,

$$(G9) \quad \forall (x, y) \in Z \setminus \{0\} (x \notin N \Rightarrow \beta(x, y, \bar{p}) < \alpha_M)$$

ensures that the production turnpike is unique – that is presented in the following theorem.

□ **Theorem 4.** In a Gale economy satisfying conditions (G1)–(G9) there a unique production turnpike exists.

Proof. The existence of a turnpike N is ensured by conditions (G1)–(G8). Let us suppose that N' is a turnpike different from N . Then $N \cap N' \neq \emptyset$ (since if $N \cap N' = \emptyset$, then $N = N'$). By the definition of the turnpike N :

$$\exists (\bar{x}, \bar{y}) \in Z \left(\alpha_M \bar{x} = \bar{y} > 0 \wedge \frac{\bar{x}}{\|\bar{x}\|} = \bar{s} \right).$$

⁴ $\langle a, b \rangle$ denotes the scalar product of vectors $a, b \in R^{2n}$: $\langle a, b \rangle = \sum_i a_i b_i$.

Similarly for the turnpike N' :

$$\exists(\bar{x}', \bar{y}') \in Z \left(\alpha_M \bar{x}' = \bar{y}' > 0 \wedge \frac{\bar{x}'}{\|\bar{x}'\|} = \bar{s}' \neq \bar{s} \right),$$

so

$$\langle \bar{p}, \bar{y}' \rangle = \alpha_M \langle \bar{p}, \bar{x}' \rangle. \quad (*)$$

By **(G9)** we get

$$\beta(\bar{x}', \bar{y}', \bar{p}) = \frac{\langle \bar{p}, \bar{y}' \rangle}{\langle \bar{p}, \bar{x}' \rangle} < \alpha_M,$$

i.e. $\langle \bar{p}, \bar{y}' \rangle < \alpha_M \langle \bar{p}, \bar{x}' \rangle$, which contradicts with (*). The contradiction terminates the proof. ■

A next consequence of condition **(G9)** (and the other conditions) is presented in the theorem below (in literature it is known as Radner's Lemma).

□ **Theorem 5.** If conditions **(G1)**–**(G9)** are satisfied, then

$$\forall \varepsilon > 0 \exists \delta_\varepsilon > 0 \forall (x, y) \in Z \left(\left\| \frac{x}{\|x\|} - \bar{s} \right\| \geq \varepsilon \Rightarrow \beta(x, y, \bar{p}) = \frac{\langle \bar{p}, y \rangle}{\langle \bar{p}, x \rangle} \leq \alpha_M - \delta_\varepsilon \right). \quad (6)$$

Proof. See Radner [1961]; Takayama [1985; Ch. 7]; Panek [2003; Ch. 5, Lemma 5.2]. ■

Theorem 5 states that the economic efficiency rate and the technological efficiency rate are equal on the turnpike only. Off the turnpike, the economic efficiency rate is less than its maximal value, which applies only at the turnpike.

2. Dynamics. “Weak” turnpike effect in a non-stationary Gale economy under a special growth condition

Let us fix any time period $t_1 < +\infty$. The set $T = \{0, 1, \dots, t_1\}$ is called a horizon (of the functioning) of the economy. The period t_1 is also the length of the economy's horizon T . Let $(x(t), y(t)) \in Z(t)$, $t \in T$. In a Gale economy the next period's inputs originate from the directly preceding period:⁵

$$x(t+1) \leq y(t), \quad t = 0, 1, \dots, t_1 - 1,$$

⁵ In this sense the economy is closed, see Gale [1956].

which due to **(G5)** leads to:

$$(y(t), y(t+1)) \in Z(t), \quad t = 0, 1, \dots, t_1 - 1. \quad (7)$$

Let us fix an initial output vector y^0 :

$$y(0) = y^0 \geq 0. \quad (8)$$

A sequence of output vectors $\{y(t)\}_{t=0}^{t_1}$ satisfying conditions (7)–(8) is said to be a (y^0, t_1) – feasible growth process in a non-stationary Gale economy. It is clear that if conditions **(G1)**–**(G6)** hold, then $\forall y^0 \geq 0 \forall t_1 < +\infty$ exist there (y^0, t_1) – feasible growth processes.

In the work of Panek [2013b] we were interested in solving the following dynamic programming problem – the maximization of value of the last period's t_1 output (at von Neumann prices):

$$\max \langle \bar{p}, y(t_1) \rangle$$

subject to: (7)–(8).

In the current paper we will deal with the properties of growth processes solving the problem of maximization of the value of output (again at von Neumann prices) throughout the whole horizon T :⁶

$$\max \sum_{t=0}^{t_1} \langle \bar{p}, y(t) \rangle \quad (9)$$

subject to: (7)–(8). (10)

The problem has a solution which we denote by $\{y^*(t)\}_{t=0}^{t_1}$ and call (y^0, t_1, \bar{p}) – the optimal growth process in a non-stationary Gale economy.

The last assumption essential for the proof of turnpike properties of optimal growth processes in the light of the criterion (9) states that there a feasible growth process exists starting from the initial state (8) and reaching the turnpike in a finite time:

(G10) There exists (y^0, \tilde{t}) – feasible growth process $\{\tilde{y}(t)\}_{t=0}^{\tilde{t}}$, where $\tilde{t} < t_1$, such that

$$\alpha(\tilde{y}(\tilde{t}), \tilde{y}(\tilde{t}+1)) = \alpha_M. \quad (11)$$

⁶ A similar problem in a non-stationary von Neumann economy was formulated and solved in Panek [2013a].

⁷ The condition is not needed if the initial output y^0 in (8) is positive.

□ **Theorem 6** (A „weak” turnpike theorem).

If in a non-stationary Gale economy with limit technology satisfying conditions (G1)–(G10) the optimal turnpike growth rate α_M meets the inequality $\alpha_M > 1$, then for any $\varepsilon > 0$ there exists a natural number k_ε , such that the number of periods in which an (y^0, t_1, \bar{p}) – optimal growth process $\{y^*(t)\}_{t=0}^{t_1}$ satisfies the inequality

$$\left\| \frac{y^*(t)}{\|y^*(t)\|} - \bar{s} \right\| \geq \varepsilon \quad (12)$$

is not greater than k_ε . The number k_ε is independent of the horizon T length.

Proof. Let us be given a (y^0, t_1, \bar{p}) – optimal growth process $\{y^*(t)\}_{t=0}^{t_1}$ which is a solution of the problem (9)–(10). Then, according to (5), (7), (G7):

$$\langle \bar{p}, y^*(t+1) \rangle \leq \alpha_M \langle \bar{p}, y^*(t) \rangle, \quad t = 0, 1, \dots, t_1 - 1. \quad (13)$$

Suppose that for an $\varepsilon > 0$ in periods $\tau_1 < \tau_2 < \dots < \tau_k$ ($0 \leq \tau_i < t_1$, $i = 1, \dots, k$) condition (12) is satisfied. Then due to Theorem 5 (condition (6)):

$$\langle \bar{p}, y^*(t+1) \rangle \leq (\alpha_M - \delta_\varepsilon) \langle \bar{p}, y^*(t) \rangle, \quad t = \tau_1, \dots, \tau_k. \quad (14)$$

From (13) and (14) we obtain an upper bound for the criterion (9) (value of production throughout the horizon T):

$$\sum_{t=0}^{t_1} \langle \bar{p}, y^*(t) \rangle \leq \langle \bar{p}, y^0 \rangle \left[\sum_{t=0}^{t_1-k} \alpha_M^t + \alpha_M^{t_1-k} \sum_{\tau=1}^k (\alpha_M - \delta_\varepsilon)^\tau \right]. \quad (15)$$

By (G10) there is a period $\tilde{t} < t_1$ and a (y^0, \tilde{t}) – feasible growth process $\{\tilde{y}(t)\}_{t=0}^{\tilde{t}}$ such that $\alpha(\tilde{y}(\tilde{t}), \tilde{y}(\tilde{t}+1)) = \alpha_M$. From the fact that $(\tilde{y}(\tilde{t}), \tilde{y}(\tilde{t}+1)) \in Z(\tilde{t}+1)$, and due to (11) and (G5)

$$(\tilde{y}(\tilde{t}), \alpha_M \tilde{y}(\tilde{t})) \in Z(\tilde{t}+1),$$

which by Theorem 4 entails that $\tilde{y}(\tilde{t}) \in N$, and thus $\tilde{y}(\tilde{t}) = \sigma \bar{s}$ for some $\sigma > 0$. By the condition (G7) we get a (y^0, t_1) – feasible growth process $\{y(t)\}_{t=0}^{t_1}$:

$$y(t) = \begin{cases} \tilde{y}(t), & t = 0, 1, \dots, \tilde{t}, \\ \sigma \bar{s} \alpha_M^{t-\tilde{t}}, & t = \tilde{t} + 1, \dots, t_1, \end{cases}$$

and we can bound the growth criterion (9) below by:

$$\sum_{t=0}^{t_1} \langle \bar{p}, y^*(t) \rangle \geq C_1 + \sigma \langle \bar{p}, \bar{s} \rangle \sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}}, \quad (16)$$

where $C_1 = \langle \bar{p}, y^0 \rangle + \sum_{t=1}^{\bar{t}} \langle \bar{p}, \tilde{y}(t) \rangle > 0$. The constant C_1 is independent of t_1 . From (15) and (16) we obtain the inequality:

$$\sigma \langle \bar{p}, \bar{s} \rangle \sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}} \langle \bar{p}, y^0 \rangle \left[\sum_{t=1}^{t_1-k} \alpha_M^t + \alpha_M^{t_1-k} \sum_{\tau=1}^k (\alpha_M - \delta_\varepsilon)^\tau \right],$$

hence

$$\sum_{t=1}^{t_1-k} \alpha_M^t + \alpha_M^{t_1-k} \sum_{\tau=1}^k (\alpha_M - \delta_\varepsilon)^\tau > \frac{\sigma \langle \bar{p}, \bar{s} \rangle}{\langle \bar{p}, y^0 \rangle} \sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}} > 0,$$

and finally

$$\alpha_M \frac{\alpha_M^{t_1-k}}{\alpha_M - 1} + \alpha_M^{t_1-k} (\alpha_M - \delta_\varepsilon) \frac{(\alpha_M - \delta_\varepsilon)^k - 1}{\alpha_M - \delta_\varepsilon - 1} > \frac{\alpha_M^{t_1-\bar{t}}}{\alpha_M - 1} \frac{\sigma \langle \bar{p}, \bar{s} \rangle}{\langle \bar{p}, y^0 \rangle}.$$

Taking a sufficiently small $\delta_\varepsilon > 0$ for which it holds $\alpha_M - \delta_\varepsilon - 1 > 0$, after simple manipulations we arrive at the following condition:

$$\forall t_1 \geq k \left(A(t_1, k) + B(t_1, k) > C_2 = \frac{\sigma \langle \bar{p}, \bar{s} \rangle}{\langle \bar{p}, y^0 \rangle} > 0 \right), \quad (17)$$

where

$$A(t_1, k) = \frac{\alpha_M (\alpha_M^{t_1-k} - 1)}{(\alpha_M^{t_1-\bar{t}+1} - 1)}, \quad B(t_1, k) = \frac{\alpha_M^{t_1-k} (\alpha_M - \delta_\varepsilon) (\alpha_M - 1) [(\alpha_M - \delta_\varepsilon)^k - 1]}{(\alpha_M^{t_1-\bar{t}} - 1) (\alpha_M - \delta_\varepsilon - 1)}.$$

Due to the facts,

$$A(t_1, k) \rightarrow 0 \quad \text{as} \quad k \rightarrow +\infty \quad (t_1 \geq k)$$

and

$$B(t_1, k) \rightarrow 0 \quad \text{as} \quad k \rightarrow +\infty \quad (t_1 \geq k),$$

we conclude that the number k in (17) is bounded. To be more specific a natural number k_ε exists (depending on ε but not on the horizon $T = \{0, 1, \dots, t_1\}$

length) such that the number of periods k , for which the inequality (12) is satisfied is not greater than k_ε . ■

3. “Very strong” turnpike effect

A simple consequence of the „weak” turnpike theorem in a non-stationary Gale economy with limit technology under the growth criterion (9) is Theorem 7.

□ **Theorem 7** (A „very strong” turnpike theorem).

If conditions **(G1)**–**(G10)** are satisfied and a (y^0, t_1, \bar{p}) – optimal growth process $\{y^*(t)\}_{t=0}^{t_1}$ reaches the turnpike N in a period $\bar{t} < t_1$, i.e.

$$\alpha(y^*(\bar{t}), y^*(\bar{t}+1)) = \alpha_M, \quad (18)$$

then $\forall t \in \{\bar{t}+1, \dots, t_1-1\} (y^*(t) \in N)$.

Proof.⁸ By (18), $y^*(\bar{t}) \in N$, i.e. $y^*(\bar{t}) = \sigma \bar{s}$ for some $\sigma > 0$. The optimal process $\{y^*(t)\}_{t=0}^{t_1}$ meets the condition (13), hence

$$\begin{aligned} \sum_{t=0}^{t_1} \langle \bar{p}, y^*(t) \rangle &= \sum_{t=0}^{\bar{t}} \langle \bar{p}, y^*(t) \rangle + \sum_{t=\bar{t}+1}^{t_1} \langle \bar{p}, y^*(t) \rangle \leq \\ &\leq \sum_{t=0}^{\bar{t}} \langle \bar{p}, y^*(t) \rangle + \sigma \langle \bar{p}, \bar{s} \rangle \sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}}. \end{aligned} \quad (19)$$

Assume that $y^*(\tau) \notin N$ for some period $\tau \in \{\bar{t}+1, \dots, t_1-1\}$, then, according to Theorem 5 (condition (6)), there is $\delta_\varepsilon > 0$ such that

$$\langle \bar{p}, y^*(\tau+1) \rangle \leq (\alpha_M - \delta_\varepsilon) \langle \bar{p}, y^*(\tau) \rangle,$$

which together with (19) leads to the inequality:

$$\sum_{t=0}^{t_1} \langle \bar{p}, y^*(t) \rangle \leq \sum_{t=0}^{\bar{t}} \langle \bar{p}, y^*(t) \rangle + \sigma \langle \bar{p}, \bar{s} \rangle \left(\sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}} - \delta_\varepsilon \alpha_M^{t-\bar{t}} \right). \quad (20)$$

On the other hand, a process $\{\tilde{y}(t)\}_{t=0}^{\bar{t}}$ of form:

$$\tilde{y}(t) = \begin{cases} y^*(t), & t = 0, 1, \dots, \bar{t}, \\ \sigma \bar{s} \alpha_M^{t-\bar{t}}, & t = \bar{t} + 1, \dots, t_1, \end{cases}$$

⁸ This proof mimicks the proof of Theorem 2 in Panek and Runka [2011].

is (y^0, t_1) – feasible, so

$$\begin{aligned} \sum_{t=0}^{t_1} \langle \bar{p}, y^*(t) \rangle &= \sum_{t=0}^{\bar{t}} \langle \bar{p}, y^*(t) \rangle + \sum_{t=\bar{t}+1}^{t_1} \langle \bar{p}, y^*(t) \rangle \geq \\ &\geq \sum_{t=0}^{\bar{t}} \langle \bar{p}, \tilde{y}(t) \rangle = \sum_{t=0}^{\bar{t}} \langle \bar{p}, y^*(t) \rangle + \sigma \langle \bar{p}, \bar{s} \rangle \sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}}. \end{aligned} \quad (21)$$

From (20) and (21) we obtain,

$$\sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}} - \delta_\varepsilon \alpha_M^{\tau-\bar{t}} \geq \sum_{t=\bar{t}+1}^{t_1} \alpha_M^{t-\bar{t}},$$

from which it follows that $\delta_\varepsilon \leq 0$. The contradictions terminate the proof. ■

Final comments

The results contained in Theorems 6 and similar ones contained in earlier papers, e.g. in Panek and Runka [2011], can be easily generalized. They are valid also in the case where the growth criterion (9) is replaced with one of the following:

$$(I) \quad \max \sum_{t=0}^{t_1} u(y(t)),$$

$u: R_+^n \rightarrow R_+^1$ is an increasing continuous homogenous of degree 1 of the utility function, positive on the turnpike such that for a positive number a it holds;

$$\forall y \geq 0 \left(u(y) \leq a \langle \bar{p}, y \rangle \right),$$

or

$$(II) \quad \max U(y(0), y(1), \dots, y(t_1)),$$

U the composition of an increasing continuous function $g: R_+^1 \rightarrow R_+^1$ and the linear function $\left\langle \bar{p}, \sum_{t=0}^{t_1} y(t) \right\rangle$:

$$U(y(0), y(1), \dots, y(t_1)) = g \left(\left\langle \bar{p}, \sum_{t=0}^{t_1} y(t) \right\rangle \right).$$

In the literature devoted to research on the properties of optimal growth processes in stationary versions of a Gale economy (under constant technology) a ‘strong’ version of turnpike property (theorems) is also mentioned, which is a kind of link between the weak and the very strong turnpike theorems. If a strong turnpike property result holds in the model we presented needs to be further investigated.

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Aims and Scope

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