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Paweł KLIBER, Krzysztof MALAGA
The Poznań University of Economics

Convergence of regional growth paths towards stable steady-states in Poland in years 1998–2000

Abstract. In the paper we present two neoclassical growth models of Solow-Swan type: with regional budget deficit and without it. The main aim of the paper is to analyze the convergence of regions in Poland towards their stable steady-states and to check the speed of this convergence.

We use the method of calibration of parameters in models and numerical methods for calculating capital and output per worker in stable steady-states. The computations were made for the new administrative division of Poland. On the base of empirical results we make conclusions about future distribution of wealth among regions and about potential possibilities of growth in regions.

We also try to answer the question if in the future there will be convergence or divergence welfare among the regions of Poland.

Keywords: Economic growth, convergence, regions, growth models of Solow-Swan type.

JEL Codes: O41, O47, R11.

1. Introduction

In the article we consider two neoclassical growth models of Solow-Swan type. One of these models takes budget deficit into consideration while the other does not[1]. We use these models to analyze paths of growth in the new Polish regions towards their stable steady-state. In particular we are interested in the speed of convergence and the length of the period of half-convergence. We also calculate the values of capital and output per worker in the steady-states for each region of Poland[2]. Based

[1] The Solow-Swan model with budget deficit was proposed by Ch. Tavera and I. Codoret in [13] and its aim was the analysis of the convergence of European countries.

[2] For the discussion about the application of growth models of Solow-Swan type in the analysis of regional growth see Jones (1997a) and Jones (1997b). See also papres on the website http://www.dse.unive.it/~smagrini.

on these empirical results we try to formulate conclusions how the distribution of wealth in Poland will change in comparison to the years 1998–2000[3].

2. Model of growth of Solow-Swan type with regional budget deficit

We consider economy of the region i where the equilibrium on the product market is given by the equation

$$Y_i(t) = C_i(t) + I_i(t) + G_i(t)\,, \tag{1}$$

where $i = 1, \ldots, 16$ stands for the number of the regions (provinces) of Poland, $Y_i(t)$ is gross product of the region i at the moment t, $C_i(t)$ is aggregate consumption in the region i in time t, $I_i(t)$ – investments in the region i in time t and $G_i(t)$ means public purchases in the region i at the moment t. We assume that aggregate consumption and savings in the region are proportional to the real income after taxation, that is:

$$C_i(t) = \tilde{c}_i\left(Y_i(t) - T_i(t)\right), \tag{2}$$

$$S_i(t) = \tilde{s}_i\left(Y_i(t) - T_i(t)\right), \tag{3}$$

where $T_i(t)$ means taxes collected in the region i in time t, $S_i(t)$ – savings in the region i in time t, $\tilde{s}_i \in [0,1]$ is the saving ratio in the region i while $\tilde{c}_i \in [0,1]$ is the consumption ratio in this region. It is assumed that savings and consumption ratios in each region are constant over time. We have

$$\tilde{s}_i + \tilde{c}_i = 1. \tag{4}$$

The budget deficit in the region i is the difference between public purchases and taxes collected in the region:

$$D_i(t) = I_i(t) + T_i(t). \tag{5}$$

[3] New administrative division of Poland divides the country into 16 regions (provinces). It has been in force since 1998 therefore there is a problem in collecting long time series. It makes using the econometric methods (like in Baumount, Cem and Gallo (2000) Canova and Marcet (1995), Caselli, Esquivel and Lefort (1996), De La Fuente (1997a) or De La Fuente (1997b)) impossible.

It comes out from the equations (2)–(5) that savings in the region cover the budget deficit and investment:

$$S_i(t) = I_i(t) + D_i(t). \tag{6}$$

Net increase in the capital stock equals gross investment less depreciation. We describe it in following differential equation:

$$\frac{dK_i(t)}{dt} = I_i(t) - \rho K_i(t), \tag{7}$$

where ρ is the rate of depreciation of physical capital. Output in the economy depends on two factors: physical capital and labor. Thus in each region we have production function of the form:

$$Y_i(t) = F_i\left(K_i(t), N_i(t)\right), \tag{8}$$

where $N_i(t)$ stands for the number of employees in the region i in time t. We assume further that production function F_i is neoclassical: it is twice-differentiable, increasing, homogenous of degree one, concave and satisfies Inada conditions. We assume that the number of the employees $N_i(t)$ grows at the constant rate η_i. That is we have the following differential equation:

$$\frac{1}{N_i(t)}\frac{dN_i(t)}{dt} = \eta_i. \tag{9}$$

From the equations (1)–(9) we can construct the following equation of capital dynamics:

$$\begin{aligned}\frac{dK_i(t)}{dt} &= \tilde{s}_i F_i\left(K_i(t), N_i(t)\right) - \left(D_i(t) + \tilde{s}_i T_i(t)\right) - \rho K_i(t) = \\ &= \tilde{s}_i F_i\left(K_i(t), N_i(t)\right) - \Delta_i(t) - \rho K_i(t),\end{aligned} \tag{10}$$

where $\Delta_i(t) = D_i(t) + \tilde{s}_i T_i(t)$ is the budget deficit in the region plus the part of savings designed for taxes.

We now consider the model with all variables expressed per worker (p.w.). The changes in capital per worker are given by the formula:

$$\frac{dk_i(t)}{dt} = \tilde{s}_i f_i\left(k(t)\right) - \delta_i - (\eta_i + \rho)k_i(t)\,, \tag{11}$$

where $y_i(t) = f_i\left(k_i(t)\right)$ is output p.w. in the region i in time t, $k_i(t) = \dfrac{K_i(t)}{N_i(t)}$ is physical capital p.w. in the region i in time t and $\delta_i = \dfrac{\Delta_i(t)}{N_i(t)}$ is the augmented budget deficit in the region i. As for the last value, we assume that it is constant over time. Figure 1 illustrates the steady-states in the Solow-Swan model we have

$$\left.\frac{dk_i(t)}{dt}\right|_{k_i(t)=\overset{*}{k}} = 0,$$

that is, by equation (11):

$$\tilde{s}_i f_i\left(\overset{*}{k}\right) = \left(\eta_i + \rho\right)\overset{*}{k} + \delta_i. \qquad (12)$$

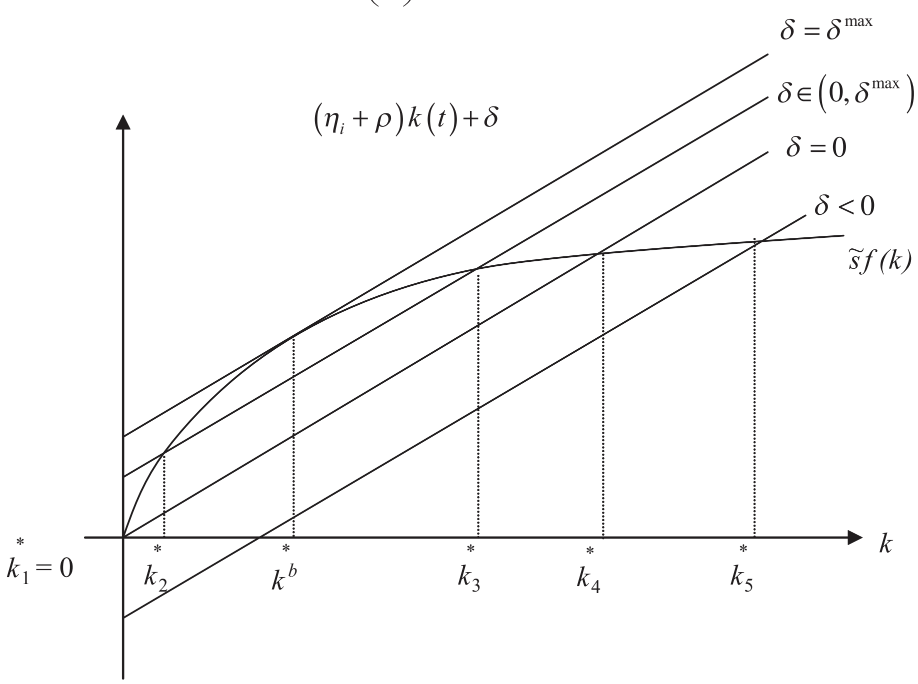

Figure 1. Steady-states in the neoclassical growth model with the budget deficit

One can easily show that steady-states $\overset{*}{k}_1$ and $\overset{*}{k}_2$ are unstable while steady-states $\overset{*}{k}_3$, $\overset{*}{k}_4$ and $\overset{*}{k}_5$ are stable. Moreover, as one can see in the figure, when the value of parameter δ increases then both steady-states in the model vanishes (for

$\delta>\delta^{\max}$) and for $\delta=\delta^{\max}$ there is the bifurcation point at $\overset{*}{k^b}$. In the case of Cobb-Douglas production function, $f_i\left(k_i(t)\right)=A_i k_i\,(t)^{\mu_i}$, we have the following equation that describes the ratio of capital p.w. growth:

$$\gamma_{k_i(t)}=\frac{\frac{dk_i(t)}{dt}}{k_i(t)}=\tilde{s}_i A_i k_i(t)^{\mu_i-1}-\frac{\delta_i}{k_i(t)}-(\eta_i+\rho). \tag{13}$$

The log-linear approximation of the equation (13) around the steady-state yields the formula (14):

$$\begin{aligned}\gamma_{k_{i(t)}}&=\underbrace{\tilde{s}_i A_i e^{-(1-\mu_i)\ln\overset{*}{k_i}}-\delta_i e^{-\ln\overset{*}{k_i}}-\eta_i-\rho}_{0}+\\&+\left((\mu_i-1)\tilde{s}_i A_i e^{-(1-\mu_i)\ln\overset{*}{k_i}}+\delta_i e^{-\ln\overset{*}{k_i}}\right)\left(\ln k_i(t)-\ln\overset{*}{k_i}\right).\end{aligned} \tag{14}$$

Using the fact that in the steady-state $\tilde{s}_i A_i \overset{*}{k_i}{}^{\mu_i-1}=\frac{\delta_i}{\overset{*}{k_i}}+\eta_i+\rho$, we obtain

$$\gamma_{k_{i(t)}}=-\left[(1-\mu_i)(\eta_i+\rho)-\mu_i\frac{\delta_i}{\overset{*}{k_i}}\right]\left(\ln k_i(t)-\ln\overset{*}{k_i}\right), \tag{15}$$

where

$$\overset{*}{k_j}=\begin{cases}\overset{*}{k_3}, & \text{if} \quad \delta\in\left(0,\delta^{\max}\right),\\ \overset{*}{k_4}, & \text{if} \quad \delta=0,\\ \overset{*}{k_5}, & \text{if} \quad \delta<0,\end{cases} \tag{16}$$

is the capital p.w. in the stable steady-state. We can now define the measure of the speed of convergence of the economy towards its stable steady-state. It is given by the following equation:

$$\beta_i^{TAV} = \frac{-\gamma_{k_i(t)}}{\ln \frac{k_i(t)}{\overset{*}{k_i}}} = \left(1-\mu_i\right)\left(\eta_i+\rho\right)-\mu_i\frac{\delta_i}{\overset{*}{k_i}}. \tag{17}$$

The parameter β_i^{TAV} says how much of the gap between the stable steady-state and the current level of capital vanishes in one time period. For example, if the value of this parameter is 0.05 it means that the economy of the region grows in one year 5% of its distance to the stable steady-state. As we can see from the equation (16), the speed of convergence increases with the real depreciation ratio of physical capital ($\eta_i + \rho$) and decreases with the elasticity of capital μ_i and with the augmented budget deficit p.w. δ_i. If the case when budget deficit is equal to zero, the formula (16) reduces to

$$\beta_i^{SOL} = \left(1-\mu_i\right)\left(\eta_i+\rho\right), \tag{18}$$

which is the speed of convergence (the so called β-coefficient) in the neoclassical Solow-Swan growth model. Solving the differential equation (14) we can calculate the time of half-convergence for the region i:

$$t_i^I = \frac{\ln 2}{\beta_i^I} \quad \text{for } I = TAV, SOL. \tag{19}$$

It gives us the number of years in which the distance to the stable steady-state reduces by half.

3. Empirical analysis of the convergence of Polish regions towards their stable steady-states

Based on the values of the parameters of production function and of growth models we can formulate some observations[4]. The share of costs of labor in the total GDP in Poland was $\frac{wN(t)}{Y(t)} = 46,2\%$ and it was relatively low in comparison to other

[4] See Kliber and Malaga (2002).

OECD countries in the years 1998-2000[5]. In half of the Polish regions (DOL, KUJ, LUS, OPL, POM, ŚLĄ, WIE and ZAH)[6] this share was smaller than in the whole Poland while in the other regions this value was greater (LUL, ŁÓD, MAŁ, MAZ, PKR, PDL, ŚWI and WRM). Real values of augmented regional budget deficit p.w. were always lower than the maximal values (the values for which there is a bifurcation point in the model). Thus there were two steady-states for each region: the stable one and the unstable one. The growth ratio of the number of employees in Poland in the years 1998-2000 was negative and it was equal to –0.2%. However, in the following regions of Poland the growth ratio of the number of employees was

Table 1. The values of parameters of production function and of the model in the regions of Poland in 1998–2000[6]

Years	Parameters	POL	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
1998–2000	A	90,5	51,4	79,2	789,7	48,6	133,6	194,0	144,0	72,2
1998–2000	μ	0,5382	**0,5956**	**0,5545**	0,3167	**0,5966**	0,4992	0,4655	0,5079	**0,5454**
1998	δ	486,6	282,6	209,7	348,4	249,3	322,0	556,0	345,3	318,2
1998	δ^{max}	4 922	8 393	2 988	2 176	5 363	3 091	3 514	11 231	3 822
1998–2000	$\eta + \rho$[7]	0,0484	**0,0502**	0,0483	**0,0507**	0,0453	**0,0535**	**0,0507**	0,0473	0,0473
1998	$\tilde{s}$	0,2223	0,2409	0,1689	0,1728	0,2178	0,1786	0,2066	0,3007	0,2222
Years	Parameters	POL	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
1998–2000	A	90,5	585,6	340,7	67,4	78,8	559,3	86,6	80,8	44,2
1998–2000	μ	0,5382	0,3504	0,3999	**0,5651**	**0,5600**	0,3551	0,5340	**0,5541**	**0,6039**
1998	δ	355,2	302,1	241,2	383,0	321,3	290,2	261,1	327,6	361,5
1998	δ^{max}	4 922	3 381	2 546	4 108	5 691	3 315	2 280	4 749	4 338
1998–2000	$\eta + \rho$	0,0498	**0,0520**	0,0489	**0,0537**	0,0457	0,0484	**0,0521**	**0,0519**	**0,0508**
1998	$\tilde{s}$	0,2223	**0,2271**	0,1903	0,2103	0,2011	0,2180	0,1746	0,2083	0,2018

[5] The abbreviations for the Polish regions (provinces): DOL – Dolnośląskie, KUJ – Kujawsko--Pomorskie, LUL – Lubelskie, LUS – Lubuskie, ŁÓD – Łódzkie, MAŁ – Małopolskie, MAZ – Mazowieckie, OPL – Opolskie, PKR – Podkarpackie, PDL – Podlaskie, POM – Pomorskie, ŚLĄ – Śląskie, ŚWI – Świętokrzyskie, WRM – Warmińsko-Mazurskie, WIE – Wielkopolskie, ZAH – Zachodniopomorskie, POL – Poland. See the figures 2–4 in the Appendix 3.

[6] The methods of calibrations of the models are given in the Appendix 2.

[7] We take $\rho = 0,05$ as the ratio of depreciation of physical capital.

Years	Para-meters	POL	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
1998–2000	A	90,5	51,9	79,2	789,7	48,6	133,6	194,0	144,0	72,2
1998–2000	μ	0,5382	**0,5956**	**0,5545**	0,3167	**0,5966**	0,4992	0,4655	0,5079	**0,5454**
1999	δ	450,6	372,1	363,9	463,7	374,9	349,0	322,7	437,8	386,3
1999	δ^{max}	5 244	9 223	3 364	2 400	6 114	3 645	3 571	12 365	3 963
1998–2000	$\eta + \rho$	0,0498	0,0484	**0,0502**	0,0483	**0,0507**	0,0453	**0,0535**	**0,0507**	0,0473
1999	$\tilde{s}$	0,2289	**0,2503**	0,1781	0,1847	**0,2297**	0,1940	0,2084	**0,3152**	0,2259
Years	Para-meters	POL	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
1998–2000	A	90,5	585,6	340,7	67,4	78,8	559,3	86,6	80,8	44,2
1998–2000	μ	0,5382	0,3504	0,3999	**0,5651**	**0,5600**	0,3551	0,5340	**0,5541**	**0,6039**
1999	δ	450,6	372,1	363,9	463,7	374,9	349,0	322,7	437,8	386,3
1999	δ^{max}	5 244	2 717	2 523	4 499	6 167	2 962	1 954	5 416	3 423
1998–2000	$\eta + \rho$	0,0498	**0,0520**	0,0489	**0,0537**	0,0457	0,0484	**0,0521**	**0,0519**	**0,0508**
1999	$\tilde{s}$	0,2289	0,1970	0,1893	0,2187	0,2083	0,2027	0,1625	0,2209	0,1838
Years	Para-meters	POL	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
1998–2000	A	90,5	51,9	79,2	789,7	48,6	133,6	194,0	144,0	72,2
1998–2000	μ	0,5382	**0,5956**	**0,5545**	0,3167	**0,5966**	0,4992	0,4655	0,5079	**0,5454**
2000	δ	493,4	537,8	394,4	333,2	411,5	357,2	432,4	911,3	292,1
2000	δ^{max}	4 831	5 958	2 701	2 178	3 542	3 075	3 505	14 122	2 441
1998–2000	$\eta + \rho$	0,0498	0,0484	**0,0502**	0,0483	**0,0507**	0,0453	**0,0535**	**0,0507**	0,0473
2000	$\tilde{s}$	0,2204	0,2098	0,1615	0,1729	0,1843	0,1782	0,2063	**0,3365**	0,1812
Years	Para-meters	POL	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
1998–2000	A	90,5	585,6	340,7	67,4	78,8	559,3	86,6	80,8	44,2
1998–2000	μ	0,5382	0,3504	0,3999	**0,5651**	**0,5600**	0,3551	0,5340	**0,5541**	**0,6039**
2000	δ	493,4	345,7	344,7	636,9	377,5	360,6	362,8	444,6	469,4
2000	δ^{max}	4 831	2 157	1 965	7 380	4 350	2 292	1 853	4 666	3 375
1998–2000	$\eta + \rho$	0,0498	**0,0520**	0,0489	**0,0537**	0,0457	0,0484	**0,0521**	**0,0519**	**0,0508**
2000	$\tilde{s}$	0,2204	0,1696	0,1629	**0,2713**	0,1787	0,1718	0,1585	0,2067	0,1827

Source: Own computations.

positive: DOL 0,02%, LUL 0,07%, MAŁ 0,07%, ŁÓD 0,35%, PKR 0,02%, POM 0,37%, WRM 0,21%, WIE 0,19% and ZAH 0,08%. In the other regions this ratio was negative. The savings ratio in the whole Poland was 22,2% in 1998, 22,9% in 1999 and 22,1% in 2000. The region in which the value of this parameter was the highest was MAZ: 30% /1998, 32% /1999 and 34% in 2000. The lowest savings ratio were in the following regions: KUJ 16,9%/1998, 16,1%/2000, LUL 17,3%/1998, ŁÓD 17,9%/1998, 17,8%/2000 and WRM 17,5%/1998.

In Table 2 we put the values of capital p.w. and output p.w. - the current ones and their values in stable and unstable steady-states. There was a diversity among the regions of Poland in the capital stock p.w. The capital stock in Poland grew from 92 419 PLN p.w. in 1998 to 95 309 PLN p.w. in 2000. The regions with the highest stock of capital per worker were DOL, LUS, MAZ, OPL, POM, ŚLĄ, WRM and ZAH. In the other regions the value of capital p.w. was lower than the average for Poland. The only region in which the current value of capital p.w. was lower than in the bifurcation point was LUL. The values of capital in the unstable steady-states are very low. Therefore we do not pay attention to these steady-states in the rest of the paper. The regions with the highest level of capital p.w. in stable steady-states are DOL, LUS, MAZ, ŚLĄ and WIE. In the other regions the level of capital p.w. in stable steady-states is lower than the average value of this variable in Poland. There was also a diversity among the regions in the values of output p.w. In the years 1998–2000 GDP p.w. in Poland grew from 43 159 PLN to 44 665 PLN. The regions with the highest value of GDP p.w. were: MAZ 51 930 PLN/1998, 55 938 PLN/1999, 57 060 PLN/2000, ŚLĄ 48 813 PLN /1998, 51 376 PLN/2000 and ZAH 48 813 PLN/1998, 50680 PLN/1999 and 53 083 PLN /2000. The regions with the lowest values of GDP p.w. were: LUL 26 947 PLN/1998, 27 326 PLN/1999 and 27 707 PLN/2000, ŚWI 27 835 PLN/1998, 29 760 PLN/1999 and 30 394 PLN /2000 and PKR 27 658 PLN/1998, 27 908 PLN/1999 and 28 371 PLN/2000. The levels of output p.w. in the unstable steady-states were very low and therefore we do not mention them in the rest of the paper. The values of output p.w. in the stable steady-states are much higher than the current values of output p.w. The regions in which the difference between the value of output in stable steady-states and the current value of this variable is the highest are: DOL, MAZ, POM and ZAH. In some regions (for example in PKR, PDL, ŚWI, WRM and LUL) the values of output p.w. in the stable steady-states are lower than the current output p.w.

In Table 3 we present the values of parameters that describe the speed of convergence of the economies towards their steady-states. As we can see, the influence of the budget deficit on these parameters is very small. The speed of convergence (β-coefficient) towards the steady-state in Poland was 2,3% per year and the period of half-convergence was about 30–31 years. In the richer regions the speed of

[8] 1 euro in 2000 = 4,0043 PLN (Polish zloties).

Table 2. The values of physical capital p.w. and output p.w. - current values and values in steady-states (in PLN 2000[8])

Year	POL	Va-riable	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
1998	92 419	k^f	102 079	82 261	71 996	98 741	79 316	82 857	115 353	120 310
	41 099	y^f	**47 556**	40 936	26 947	**43 815**	35 232	35 269	**51 930**	**41 506**
	365 301	k^{sol}	713 253	227 596	97 832	443 399	226 885	202 374	779 234	298 693
	89 181	y^{sol}	**158 846**	73 984	30 071	**113 665**	63 042	57 279	**141 450**	69 894
	350 819	k^{tav1}	690 159	215 647	91 942	427 576	216 683	191 790	758 321	283 681
	87 261	y^{tav1}	**155 762**	71 805	29 485	**111 227**	61 611	55 865	**139 509**	67 956
	225	k^{tav2}	531	278	4	394	116	94	158	309
	1 670	y^{tav2}	2 178	1 795	1 226	1 720	1 436	1 608	1 884	1 647
	95 504	k^b	198 036	60 576	18 186	123 232	56 668	48 403	196 678	78 713
Year	POL	Va-riable	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
1998	92 419	k^f	62 289	82 245	102 064	99 367	70 511	94 484	88 481	109 390
	41 099	y^f	27 658	30 769	**46 941**	**48 304**	27 835	36 773	**43 304**	**45 813**
	365 301	k^{sol}	151 967	136 798	298 820	495 139	161 636	152 415	359 232	368 541
	89 181	y^{sol}	38 302	38 590	83 695	**121 672**	39 534	50 749	**96 712**	**101 653**
	350 819	k^{tav1}	143 668	129 155	283 352	480 112	153 030	142 486	345 832	351 613
	87 261	y^{tav1}	37 556	37 713	81 219	**119 590**	38 773	48 956	**94 696**	**98 807**
	225	k^{tav2}	11	28	391	233	12	233	233	538
	1 670	y^{tav2}	1 348	1 287	1 966	1 668	1 347	1 590	1 654	1 972
	95 504	k^b	30 244	29 703	80 432	132 559	32 454	39 662	95 573	103 158
Year	POL	Va-riable	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
1999	94 442	k^f	102 451	84 652	73 531	96 475	80 996	72 909	123 106	118 075
	43 159	y^f	**49 772**	42 084	27 326	**45 608**	38 340	35 984	**55 938**	40 646
	369 067	k^{sol}	747 119	241 680	100 509	464 416	255 876	196 341	826 968	287 303
	89 675	y^{sol}	**163 296**	76 489	30 329	**116 849**	66 943	56 478	**145 786**	68 428
	351 158	k^{tav1}	720 441	227 682	91 787	445 458	242 064	182 873	801 004	267 875
	87 306	y^{tav1}	**159 797**	74 000	29 469	**113 980**	65 114	54 640	**143 443**	65 864
	334	k^{tav2}	659	355	13	521	190	164	229	521
	440	y^{tav2}	831	595	18	739	251	184	346	320
	96 489	k^b	207 439	64 325	18 683	129 074	63 908	46 960	208 726	75 711
Year	POL	Va-riable	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
1999	94 442	k^f	62 077	81 308	103 816	108 952	71 953	95 674	88 905	115 120

	43 159	y^{f}	27 908	30 606	**49 638**	**51 376**	29 760	40 180	**43 973**	**50 680**
	369 067	k^{sol}	113 858	125 330	313 857	515 705	136 379	122 281	390 885	270 979
	89 675	y^{sol}	34 617	37 263	86 050	**124 476**	37 219	45 117	**101 344**	84 425
	351 158	k^{tav1}	103 992	114 133	295 502	498 579	126 461	110 208	373 290	253 477
	87 306	y^{tav1}	33 535	35 894	83 169	**122 144**	36 235	42 680	**98 791**	81 088
	334	k^{tav2}	30	82	513	286	24	417	358	714
	440	y^{tav2}	38	108	623	399	45	577	424	1 066
	96 489	k^{b}	22 660	27 213	84 480	138 065	27 383	31 820	103 995	75 850
Year	POL	Variable	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
2000	95 309	k^{f}	104 593	81 919	71 466	99 382	80 606	75 645	124 971	116 009
	44 665	y^{f}	**53 253**	43 781	27 707	**49 316**	39 053	38 273	**57 060**	44 242
	338 492	k^{sol}	485 933	193 916	90 096	267 582	214 206	193 021	950 739	177 464
	85 597	y^{sol}	**126 390**	67 698	29 296	84 095	61 258	56 031	**156 487**	52 617
	318 628	k^{tav1}	460 555	177 242	81 103	249 292	199 731	178 954	916 509	165 486
	82 856	y^{tav1}	**122 416**	64 405	28 337	80 617	59 155	54 092	**153 599**	50 650
	440	k^{tav2}	831	595	18	739	251	184	346	320
	2 393	y^{tav2}	2 843	2 737	1 970	2 502	2 109	2 197	2 804	1 679
	88 495	k^{b}	134 920	51 612	16 748	74 368	53 501	46 166	239 965	46 766
Year	POL	Variable	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
2000	95 309	k^{f}	61 689	79 460	107 897	108 958	70 089	100 643	90 733	115 038
	44 665	y^{f}	28 371	32 489	42 032	**54 131**	30 894	42 664	**46 967**	**53 083**
	338 492	k^{sol}	89 675	97 844	473 823	361 920	105 098	113 573	335 802	268 033
	85 597	y^{sol}	31 839	33 750	**108 600**	**102 087**	33 931	43 371	**93 164**	83 869
	318 628	k^{tav1}	80 446	87 079	449 233	344 420	94 589	99 826	317 832	246 128
	82 856	y^{tav1}	30 650	32 213	**105 379**	**99 293**	32 685	40 484	**90 367**	79 660
	440	k^{tav2}	38	108	623	399	45	577	424	1 066
	2 393	y^{tav2}	2 102	2 213	2 557	2 255	2 162	2 582	2 307	2 979
	88 495	k^{b}	17 847	21 245	127 538	96 893	21 102	29 554	89 340	75 025

k^{f} – current value of physical capital p.w. in the region or in Poland, y^{f} – current value of output (GDP) p.w. in the region or in Poland, k^{SOL} – value of physical capital p.w. in the region or in Poland in steady-state in the model without budget deficit, y^{SOL} – value of output p.w. in the region or in Poland in steady-state in the model without budget deficit, k^{tav1} – value of physical capital p.w. in the region or in Poland in stable steady-state in the model with budget deficit, y^{tav1} – value of output p.w. in the region or in Poland in stable steady-state in the model with budget deficit, k^{tav2} – value of physical capital p.w. in the region or in Poland in unstable steady-state in the model with budget deficit, y^{tav2} – value of output p.w. in the region or in Poland in unstable steady-state in the model with budget deficit, k^{b} – value of capital p.w. in the bifurcation point.

Source: Own computation.

Table 3. The speed of convergence coefficients β and the times of half-convergence in the models with and without budget deficit

Years	POL	Parameter	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
1998–2000	0,0230	β^{sol}	0,0196	**0,0224**	**0,0330**	0,0205	0,0227	**0,0286**	**0,0249**	0,0215
	0,0225	β^{tav1}	0,0192	0,0217	**0,0324**	0,0200	0,0222	**0,0279**	**0,0246**	0,0209
	30,2	t^{sol}	35,4	31,0	**21,0**	33,9	30,6	**24,3**	**27,8**	32,2
	30,8	t^{tav1}	36,1	32,0	**21,4**	34,6	31,3	**24,9**	**28,2**	33,2
Years	POL	Parameter	PKR	PDL	POM	ŚLĄ	ŚWI	WRM	WIE	ZAH
1998–2000	0,0230	β^{sol}	**0,0338**	**0,0293**	**0,0234**	0,0201	**0,0312**	**0,0243**	**0,0231**	0,0201
	0,0225	β^{tav1}	**0,0331**	**0,0287**	**0,0227**	0,0197	**0,0306**	**0,0234**	**0,0227**	0,0195
	30,2	t^{sol}	**20,5**	**23,6**	**29,7**	34,5	**22,2**	**28,5**	**29,9**	34,5
	30,8	t^{tav1}	**20,9**	**24,2**	**30,6**	35,1	**22,7**	**29,6**	**30,6**	35,5

β^{sol} – the speed of convergence towards the steady-state in the model without budget deficit,
β^{tav1} – the speed of convergence towards the stable steady-state in the model with budget deficit,
t^{sol} – the time of half-convergence towards the steady-state in the model without budget deficit,
t^{tav1} – the time of half-convergence towards the stable steady-state in the model with budget deficit.

Source: Own computation.

convergence was lower than in the poorer ones. It means that the poorer regions were closer to their stable steady-states.

Table 4 and Figures 2–4 in Appendix 3 show the relations between the output p.w. in the regions and the output p.w. in Poland. We present the values of this relation for the empirical output in the years 1998–2000 as well as for the output in stable steady-states. The results show that there will be a growth of diversity of wealth among the regions. There is a group of regions that will improve their positions. The main part of this group are regions: MAZ, DOL and WIE. There are also regions LUS/1998, ŚLĄ/1998, 1999 and POM/2000 in this group. The other regions will worsen their positions.

4. Conclusion

The application of the neoclassical growth models (with and without budget deficit) to the analysis of the growth in the Polish regions supports the logic of the models. In the relatively richer regions the speed of convergence towards their steady-states was higher than in the poorer regions. It means that the richer regions were closer to their steady-states than the poorer ones.

If we take the values of output p.w. in steady-states as the indicators of the future welfare of the regions, we can draw conclusions about the future distribution of welfare in Poland. We can conclude that there will be great differences between the regions. All the regions (except for MAZ) that lie in the east part of Poland will be relatively poorer in comparison to the rest of the country. On the other hand, five regions (DOL, MAZ, ŚLĄ, LUS, WIE) will be relatively richer.

5. Appendixes

1. The data

Table 4. More important data used in the analysis (measured in current PLN)

Year 1998	GDP	Number of emploees	Capital stock	Budget deficit
POL	653 960 837 880,0	15 912 000,0	1 470 569 566 281,0	1 621 543,5
DOL	50 381 305 507,	1 059 400,0	108 142 530 008,1	266 785,1
KUJ	232 715 762 489,	799 200,0	65 743 053 038,1	118 491,7
LUL	027 490 904 121,	1 020 200,0	73 450 094 215,5	43 350,5
LUS	915 760 342 781,	359 700,0	35 517 270 421,2	42 175,0
ŁÓD	139 978 253 006,5	1 134 700,0	89 999 476 023,3	39 930,4
MAŁ	49 429 000 731,9	1 401 500,0	116 124 594 819,9	159 616,0
MAZ	125 193 522 791,7	2 410 800,0	278 092 132 630,2	121 451,0
OPL	16 278 611 116,2	392 200,0	47 185 573 404,9	49 617,7
PKR	27 215 407 938,3	984 000,0	61 292 112 123,3	71 821,2
PDL	15 796 965 344,1	513 400,0	42 224 515 628,7	27 149,5
POM	36 421 375 178,1	775 900,0	79 191 566 995,5	163 738,3
ŚLĄ	92 507 530 902,3	1 915 100,0	190 298 634 409,8	160 021,5
ŚWI	17 335 349 264,7	622 800,0	43 914 469 705,2	35 594,2
WRM	18 974 976 851,4	516 000,0	48 753 727 925,1	38 626,3
WIE	59 889 586 097,7	1 383 000,0	122 369 332 497,9	167 518,7
ZAH	28 591 943 757,9	624 100,0	68 270 482 434,3	115 656,4
Year 1999	GDP	Number of emploees	Capital stock	Budget deficit
POL	677 242 055 400,0	15 691 700,0	1 481 955 358 500,0	1 066 078,6
DOL	53 385 067 800,0	1 072 600,0	109 889 268 600,0	180 233,7
KUJ	32 653 127 700,0	775 900,0	65 681 806 500,0	80 499,6
LUL	27 371 961 000,0	1 001 700,0	73 656 459 600,0	52 667,2
LUS	16 268 376 000,0	356 700,0	34 412 745 900,0	-8 234, 9
ŁÓD	42 419 328 000,0	1 106 400,0	89 614 463 700,0	29 314,1

MAŁ	50 431 194 900,0	1 401 500,0	102 182 158 500,0	207 211,2
MAZ	132 495 220 800,0	2 368 600,0	291 588 509 700,0	-103 273,8
OPL	15 965 711 100,0	392 800,0	46 379 735 100,0	62 411,3
PKR	27 321 975 600,0	979 000,0	60 773 548 500,0	78 934,4
PDL	15 590 710 500,0	509 400,0	41 418 298 800,0	57 941,4
POM	38 847 023 400,0	782 600,0	81 246 643 500,0	131 129,1
ŚLĄ	94 274 115 900,0	1 835 000,0	199 926 296 100,0	21 324,9
ŚWI	18 126 974 100,0	609 100,0	43 826 295 900,0	699,7
WRM	19 897 272 000,0	495 200,0	47 378 011 800,0	50 513,8
WIE	61 826 544 900,0	1 406 000,0	125 001 044 100,0	189 282,2
ZAH	30 367 451 700,0	599 200,0	68 980 072 200,0	35 424,7
Year 2000	GDP	Number of emploees	Capital stock	Budget deficit
POL	677 081 700 000,0	15 159 200,0	1 444 803 700 000,0	3 056 476,0
DOL	54 616 100 000,0	1 025 600,0	107 270 100 000,0	310 300,0
KUJ	33 392 000 000,0	762 700,0	62 479 500 000,0	237 089,8
LUL	27 125 100 000,0	979 000,0	69 964 900 000,0	87 225,0
LUS	16 279 200 000,0	330 100,0	32 805 900 000,0	49 510,6
ŁÓD	41 630 500 000,0	1 066 000,0	85 925 800 000,0	120 247,0
MAŁ	51 064 300 000,0	1 334 200,0	100 925 900 000,0	268 520,9
MAZ	136 201 600 000,0	2 387 000,0	298 304 600 000,0	714 700,0
OPL	16 458 200 000,0	372 000,0	43 155 200 000,0	-40 386,0
PKR	26 827 300 000,0	945 600,0	58 332 900 000,0	113 058,5
PDL	16 098 400 000,0	495 500,0	39 372 600 000,0	80 407,3
POM	31 116 600 000,0	740 300,	79 875 800 000,0	252 300,0
ŚLĄ	94 773 100 000,0	1 750 800,0	190 764 000 000,0	241 102,4
ŚWI	18 357 500 000,0	594 200,0	41 646 700 000,0	74 531,2
WRM	19 369 600 000,0	454 000,0	45 691 700 000,0	105 651,8
WIE	63 456 600 000,0	1 351 100,0	122 589 800 000,0	251 227,5
ZAH	30 315 600 000,0	571 100,0	65 698 300 000,0	190 990,0

Source: Statistical yearbook of the Republic of Poland, Statistical yearbook of the regions – Poland.

2. The methods of calibration of the models

1. The values of the capital p.w. and output p.w. in stable and unstable steady-states in region *i* were computed by numerical solving of the specific equations.
2. The values of capital p.w. in bifurcation points, k_i^b, were calculated from the equation: $\tilde{s}_i f'\left(k_i^b\right) = \eta + \rho$. In the case of Cobb-Douglas production function it yields the formula:

$$k_i^b = \left(\frac{\mu_i A_i \tilde{s}_i}{\eta_i + \rho} \right)^{\frac{1}{1-\mu_i}}.$$

3. The values of parameters $\delta_i^{\max}$ were calculated from the equation:

$$\delta_i^{\max} = \tilde{s} f_i \left(k_i^b \right) - \left(\eta_i + \rho \right) k_i^b = \tilde{s}_i \left(1 - \mu_i \right) k_i^{b^{\mu}} A_i.$$

4. The elasticities of the physical capital were computed from the necessity conditions of maximizing the gain by producers:

$$\Pi_i \left(K_i(t), N_i(t) \right) = \left\{ A_i K_i^{\mu_i}(t) N_i^{1-\mu_i}(t) - r_i K_i(t) - w_i N_i(t) \right\} \to \max,$$
$$K_i(t),\ N_i(t) \geq 0,$$

thus: $$(1-\mu_i) = \frac{w_i}{A_i K_i^{\mu_i}(t) N_i^{-\mu_i}(t)} = \frac{w_i N_i(t)}{A_i K_i^{\mu_i}(t) N_i^{1-\mu_i}(t)} = \frac{w_i N_i(t)}{Y_i(t)}$$

$$\mu_i = 1 - \frac{w_i N_i(t)}{Y_i(t)},$$

where w_i stands for average yearly wages in the region i.

5. The values of total productivity factor A_i in the Cobb-Douglas production function were calculated as the average from the equation:

$$A_i = \frac{y_i(t)}{k_i^{\mu_i}(t)}.$$

3. The relations of output p.w. in the regions to the output p.w. in Poland

Table 5. The relations of the output p.w. in the regions to the output p.w. in Poland (current values and values in stable steady-states).

1998	DOL	KUJ	LUL	LUS	ŁÓD	MAŁ	MAZ	OPL
y_i^f / y^f	1,157	0,996	0,656	1,066	0,857	0,858	1,264	1,010
Y_i^{SOL} / y^{SOL}	1,872	0,838	0,318	1,327	0,700	0,627	1,547	0,793
Y_i^{TAV} / y^{TAV}	1,924	0,861	0,316	1,391	0,733	0,607	1,584	0,775
1998	**PKR**	**PDL**	**POM**	**ŚLĄ**	**ŚWI**	**WRM**	**WIE**	**ZAH**

y_i^f / y^f	0,673	0,749	1,142	1,175	0,677	0,895	1,054	1,115
Y_i^{SOL} / y^{SOL}	0,409	0,416	0,955	1,363	0,423	0,583	1,079	1,186
Y_i^{TAV} / y^{TAV}	0,364	0,397	0,967	1,387	0,392	0,507	1,123	0,984
1999	**DOL**	**KUJ**	**LUL**	**LUS**	**ŁÓD**	**MAŁ**	**MAZ**	**OPL**
y_i^f / y^f	1,153	0,975	0,633	1,057	0,888	0,834	1,296	0,942
Y_i^{SOL} / y^{SOL}	1,914	0,865	0,317	1,387	0,734	0,611	1,570	0,782
Y_i^{TAV} / y^{TAV}	1,924	0,861	0,316	1,391	0,733	0,607	1,584	0,775
1999	**PKR**	**PDL**	**POM**	**ŚLĄ**	**ŚWI**	**WRM**	**WIE**	**ZAH**
y_i^f / y^f	0,647	0,709	1,150	1,190	0,690	0,931	1,019	1,174
Y_i^{SOL} / y^{SOL}	0,366	0,401	0,972	1,378	0,392	0,519	1,122	0,994
Y_i^{TAV} / y^{TAV}	0,364	0,397	0,967	1,387	0,392	0,507	1,123	0,984
2000	**DOL**	**KUJ**	**LUL**	**LUS**	**ŁÓD**	**MAŁ**	**MAZ**	**OPL**
y_i^f / y^f	1,192	0,980	0,620	1,104	0,874	0,857	1,278	0,991
Y_i^{SOL} / y^{SOL}	1,542	0,801	0,322	1,047	0,705	0,633	1,756	0,628
Y_i^{TAV} / y^{TAV}	1,545	0,790	0,321	1,041	0,703	0,631	1,776	0,624
2000	**PKR**	**PDL**	**POM**	**ŚLĄ**	**ŚWI**	**WRM**	**WIE**	**ZAH**
y_i^f / y^f	0,635	0,727	0,941	1,212	0,692	0,955	1,052	1,188
Y_i^{SOL} / y^{SOL}	0,353	0,379	1,344	1,184	0,374	0,527	1,079	1,030
Y_i^{TAV} / y^{TAV}	0,351	0,374	1,349	1,189	0,372	0,513	1,081	1,016

y_i^f / y^f – the relation of the current output p.w. in the region *i* to the output p.w. in Poland,
Y_i^{SOL} / y^{SOL} – the relation of the output p.w. in the region *i* in the steady-state in the model without budget deficit to the output p.w. in Poland in the steady-state in the same model,
Y_i^{TAV} / y^{TAV} – the relation of the output p.w. in the region *i* in the steady-state in the model with budget deficit to the output p.w. in Poland in the steady-state in the same model.

Source: Own computations.

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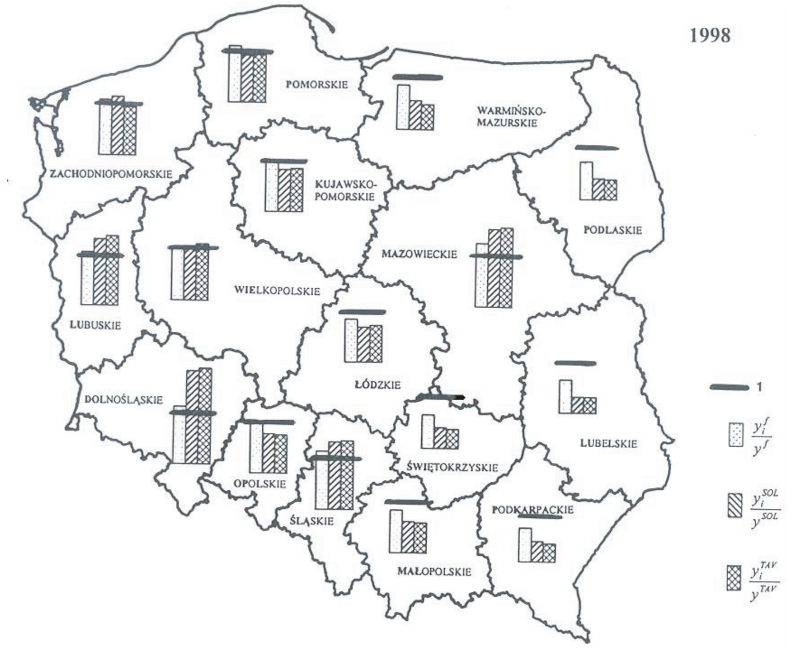

y_i^f / y^f, Y_i^{SOL} / y^{SOL}, Y_i^{TAV} / y^{TAV} – see the table 5.

Figure 2. The relations of output p.w. in regions to the output p.w. in Poland

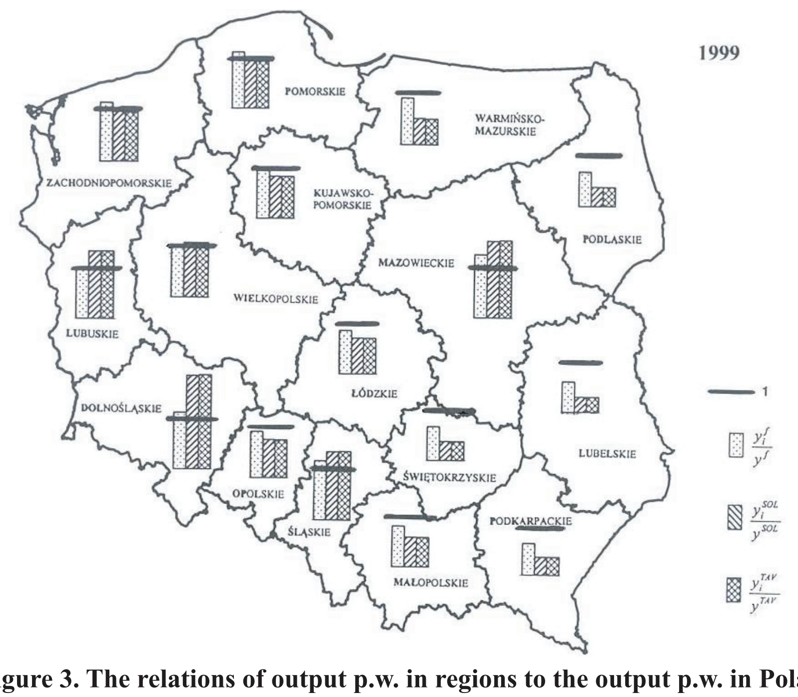

Figure 3. The relations of output p.w. in regions to the output p.w. in Poland

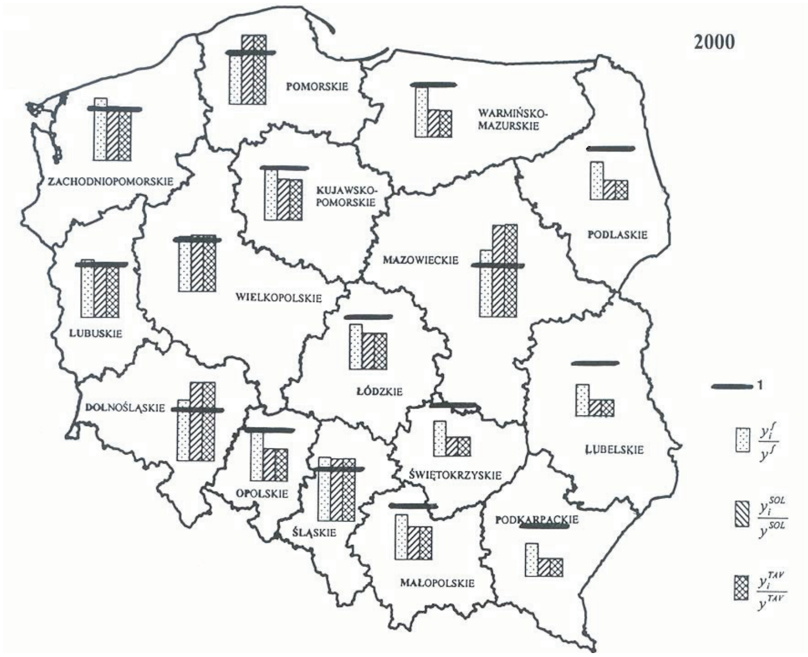

Figure 4. The relations of output p.w. in regions to the output p.w. in Poland

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