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Application of neoclassical growth models to analysis of regional inequalities in Poland

Abstract: In this paper we apply the neoclassical growth models of Solow-Swan (1956) and Mankiw-Romer-Weil (1992) to the analysis of regional inequalities. The regional inequalities in Poland are described by the values of parameters in both of the growth models, the speed of convergence of the growth paths of GDP per worker *(p.w.)* towards their steady-states and the distributions of GDP *p.w.* among the sixteen Polish regions (voivodships) in 1999 – compared with their counterparts in the steady-states. We also try to evaluate the usefulness of the neoclassical growth models for the analysis of regional inequalities in Poland.

Keywords: Growth, convergence, speed of convergence, Polish regions, neoclassical growth models, regional inequalities.

JEL codes: O41, O47, R11.

1. Introduction

The goal of this paper is to analyze regional inequalities in Poland. In the paper we strongly rely on the results of our work contained in Kliber P., Maćkowiak P., Malaga K. (2004), Kliber P., Malaga K. (2003a), (2003b), (2002) and Malaga K. (2004).

The analysis is based on the neoclassical growth models of Solow-Swan (1956) and Mankiw-Romer-Weil (1992). In the section 2 we present the appropriate growth models, definitions of stable steady-states and measures of convergence of GDP *p.w.* towards its stable steady-states. In the section 3 there are presented the methods of calibration of the models. Section 4 contains empirical results obtained for both of the growth models considered.

The methods of description of interregional inequalities – which go along with the logic of neoclassical growth models – stem also from limited availability of data on the Polish regions – which is caused by a structural reform of the Polish voivodships in 1998.

The regional inequalities in Poland are described according to the following sequence: analysis of the diversity of parameters, comparative analysis of the real values of GDP per worker with the values in steady-states, evaluation of the speed of convergence and the periods of half-convergence towards the stable steady-states in the Polish regions, analysis of distribution of the real values of GDP per worker and values in steady-states in the Polish regions in relation to GDP per worker in Poland.

At the end of the paper we conclude with consideration on the usefulness of neoclassical growth models to describe regional inequalities in Poland.

2. The neoclassical models of growth

2.1. The Solow-Swan model

We consider economy of the region *i* where the equilibrium on the product market at the moment *t* is given by the equation:

$$Y_i(t) = C_i(t) + I_{Ki}(t) \tag{1}$$

where: i=1,…,16 stands for the number of regions (voivodships) of Poland, $Y_i(t)$ – gross product of the region *i* at the moment *t* , $C_i(t)$ – aggregate consumption in the region *i* at the moment *t*, $I_{Ki}(t)$ – investments in the physical capital in the region *i* at the moment *t*. We assume that aggregate consumption and savings in the region *i* at the moment *t* are proportional to the real income:

$$C_i(t) = c_i Y_i(t),\ S_i(t) = s_{K_i} Y_i(t) \tag{2}$$

where: $S_i(t)$ – savings in the region *i* at the moment *t*, $s_{K_i} \in [0,1]$ – saving ratio in the region *i*, $c_i \in [0,1]$ – consumption ratio in the region *i*. It is assumed that savings and consumption ratios in each region are constant and $s_{K_i} + c_i = 1$. The savings are equal to the investments in physical capital in the region *i* at the moment *t:*

$$S_i(t) = I_{Ki}(t). \tag{3}$$

Net increase in the physical capital stock equals gross investment less depreciation. What is describes in the following equation:

$$\frac{dK_i(t)}{dt} = I_{K_i}(t) - \rho K_i(t) \tag{4}$$

where: ρ – the rate of depreciation of physical capital, $K_i(t)$ – the stock of the physical capital in the region i at the moment t. Output in the region i at the moment t depends on two factors: physical capital and labor. Thus in each region we have the neoclassical production function[1] of the form:

$$Y_i(t) = F_i(K_i(t), N_i(t)) = A_i K_i^{\alpha_i}(t) N_i(t)^{1-\alpha_i}, \quad \alpha_i \in (0,1) \tag{5}$$

where: A_i – the total productivity factor in the region i at the moment t, $N_i(t)$ – the number of workers in the region i at the moment t.

We assume that the number of workers $N_i(t)$ grows at the constant rate:

$$\frac{dN_i(t)}{dt}\frac{1}{N_i(t)} = \eta_i\,. \tag{6}$$

From equations (1)-(6) it follows that:

$$\frac{dK_i(t)}{dt} = s_{K_i} A_i K_i(t)^{\alpha_i} N_i(t)^{1-\alpha_i} - \rho K_i(t)\,. \tag{7}$$

Now we consider the model with all variables expressed *per worker (p.w.)*. The changes in physical capital *p.w.* are given by the formula:

$$\frac{dk_i(t)}{dt} = s_{K_i} A_i k_i(t)^{\alpha_i} - (\eta_i + \rho) k_i(t) \tag{8}$$

where: $A_i k_i(t)^{\alpha_i} = y_i(t)$ – GDP *p.w.* in the region i at the moment t, $k_i(t) = \dfrac{K_i(t)}{N_i(t)}$ – the stock of physical capital *p.w.* in the region i at the moment t.

The steady-state in the Solow-Swan model for each region is defined by the equation:

$$\left.\frac{dk_i(t)}{dt}\right|_{k_i(t) = \overset{*}{k_i}} = 0 \Leftrightarrow s_{K_i} A_i \overset{*}{k_i}(t)^{\alpha_i} = (\eta_i + \rho)\overset{*}{k_i}(t)\,. \tag{9}$$

The value of physical capital *p.w.* and GDP *p.w.* in the steady-state in the region i equals:

[1] The neoclassical production function it is twice-differentiable, increasing, homogenous of degree one, concave and satisfies Inada conditions.

$$\overset{*}{k}_i = \left(\frac{A_i s_{K_i}}{n_i + \rho} \right)^{\frac{1}{1-\alpha_i}}, \quad \overset{*}{y}_i = \left(\frac{A_i s_{K_i}}{n_i + \rho} \right)^{\frac{\alpha_i}{1-\alpha_i}}. \tag{10}$$

The rate of physical capital *p.w.* is described by the equation:

$$\gamma_{y_i(t)} = \frac{dy_i(t)}{dt}\frac{1}{y_i(t)} = \alpha_i \frac{dk_i(t)}{dt}\frac{1}{k_i(t)} = \alpha_i \gamma_{k_i(t)} \tag{11}$$

where: $\gamma_{k_i(t)} = s_{K_i} A_i k_i(t)^{\alpha_i - 1} - (\eta_i + \rho)$.

The log-linear approximation of the equation (11) around the steady-state yields the formula:

$$\gamma_{y_i(t)} \cong -\left[(1-\alpha_i)(\eta_i + \rho)\right]\left(\ln y_i(t) - \ln \overset{*}{y}_i \right). \tag{12}$$

Now we can define the measure of the speed of convergence of the growth paths of GDP *p.w.* in the region i towards its steady-state:

$$\beta_i^{SOL} = -\frac{\gamma_{k_i(t)}}{\ln \frac{k_i(t)}{\overset{*}{k}_i}} = (1-\alpha_i)(\eta_i + \rho). \tag{13}$$

The parameter β_i^{SOL} says how fast the gap between the stable steady-state and the current level of GDP *p.w.* vanishes in one period. As we can see from equation (13), the speed of convergence increases with the real depreciation rate $(\eta_i + \rho)$ and decreases with the elasticity of production with respect to physical capital.

Solving the differential equation (12) we can calculate the time of half-convergence for the region i:

$$t_i^{SOL} = \frac{\ln 2}{\beta_i^{SOL}} \tag{14}$$

it gives us the number of years in which the distance between the actual GDP *p.w.* in region i $y_i(t)$ and GDP *p.w.* in the steady-state reduces by half[2].

[2] See Barrro R. Sala-i-Martin X. (2003).

2.2. The Mankiw-Romer-Weil model

We take the assumptions (1)–(2) and we assume that the savings in region *i* at the moment *t* equal the sum of investments in human and physical capital:

$$S_i(t) = I_{Ki}(t) + I_{Hi}(t). \tag{15}$$

The dynamics of physical and human capital is given by the following system of differential equations:

$$\begin{aligned} \frac{dK_i(t)}{dt} &= I_{K_i}(t) - \rho K_i(t) = s_{K_i} Y_i(t) - \rho K_i(t) \\ \frac{dH_i(t)}{dt} &= I_{H_i}(t) - \rho H_i(t) = s_{H_i} Y_i(t) - \rho H_i(t) \end{aligned} \tag{16}$$

where: ρ – the rate of depreciation of physical capital or human capital, $I_{K_i}(t)$ – investments in physical capital in the region *i* at time *t*, $I_{H_i}(t)$ – investments in human capital in the region *i* at time *t*, s_{K_i} – the investment rate in physical capital in region *i*, s_{H_i} – the investment rate in human capital in region *i*

The production process is described by the neoclassical production function with Hicks-neutral technical progress:

$$Y_i(t) = F_i(K_i(t), N_i(t)) = A_i K_i^{\alpha_i}(t) H_i(t)^{\beta_i} N_i(t)^{1-\alpha_i-\beta_i} \tag{17}$$

where: A_i – the total productivity factor in the region *i*, $N_i(t)$ – the number of workers in the region *i* at the moment *t*, $K_i(t)$ – the stock of the physical capital in the region *i* at the moment *t*, $H_i(t)$ – the stock of the human capital in the region *i* at the moment *t*.

We assume that the number of workers in the region *i,* grows at the constant rate:

$$\frac{dN_i(t)}{dt}\frac{1}{N_i(t)} = \eta_i. \tag{18}$$

From equations (16)–(18) we can construct the following equations of physical and human capital dynamics:

$$\begin{aligned} \frac{dK_i(t)}{dt} &= s_{K_i} A_i K_i(t)^{\alpha_i} H_i(t)^{\beta_i} N_i(t)^{1-\alpha_i-\beta_i} - \rho K_i(t) \\ \frac{dH_i(t)}{dt} &= s_{H_i} A_i K_i(t)^{\alpha_i} H_i(t)^{\beta_i} N_i(t)^{1-\alpha_i-\beta_i} - \rho H_i(t). \end{aligned} \tag{19}$$

The accumulation of human capital *p.w*. and physical capital *p.w*. can be described by the following system of differential equations:

$$\frac{dk_i(t)}{dt} = s_{K_i} A_i k_i(t)^{\alpha_i} h_i(t)^{\beta_i} - (\eta_i + \rho) k_i(t),$$
$$\frac{dh_i(t)}{dt} = s_{H_i} A_i k_i(t)^{\alpha_i} h_i(t)^{\beta_i} - (\eta_i + \rho) h_i(t) \tag{20}$$

where: $y_i(t) = f_i(k_i(t), h_i(t)) = A_i k_i(t)^{\alpha_i} h_i(t)^{\beta_i}$ – GDP *p.w.* at the moment *t* in the region *i*, $k_i(t) = \frac{K_i(t)}{N_i(t)}$ – the physical capital *p.w*. in the region *i* at the moment *t*, $h_i(t) = \frac{H_i(t)}{N_i(t)}$ – the human capital *p.w.* in the region *i* at the moment *t.*

The value of GDP p.w. in steady-state for region *i* are given by the following equations:

$$\overset{*}{y_i} = \left(\frac{A_i s_{K_i}^{\alpha_i} s_{H_i}^{\beta_i}}{n_i + \rho} \right)^{\frac{1}{1 - \alpha_i - \beta_i}}. \tag{21}$$

The rate of growth of GDP p.w. in the Mankiw-Romer-Weil model with a neoclassical production function in an "intensive form": $f_i(k_i(t)) = A_i k_i(t)^{\alpha_i} h_i(t)^{\beta}$ is given by the equation:

$$\gamma_{y_i(t)} = \frac{dy_i(t)}{dt} \frac{1}{y_i(t)} = \alpha_i \frac{dk_i(t)}{dt} \frac{1}{k_i(t)} + \beta_i \frac{dh_i(t)}{dt} \frac{1}{h_i(t)}. \tag{22}$$

If we make log-linear approximation of this growth rate in the neighborhood of the steady-state we obtain the equation:

$$\gamma_{y_i(t)} \cong -\left[(1 - \alpha_i - \beta_i)(\eta_i + \rho) \right] \left(\ln y_i(t) - \ln \overset{*}{y_i} \right). \tag{23}$$

Based on this equation, we define the measure of the speed of convergence of the growth paths of GDP *p.w.* towards the steady-state in the region *i*:

$$\beta_i^{MRW} = (1 - \alpha_i - \beta_i)(\eta_i + \rho). \tag{24}$$

The speed of convergence of the growth paths of GDP *p.w.* in the region *i* towards the steady-state increases with the depreciation rate of human and physical capital and decreases with the elasticity of GDP *p.w.* with respect to human and physical capital. This coefficient describes what part of the gap between the actual GDP *p.w.* and GDP *p.w.* in the steady-state vanishes in the unit of time. Solving the differential equation (23), one can derive the following equation describing the period of half-convergence in the region *i*:

$$t_i^{MRW} = \frac{\ln 2}{\beta_i^{MRW}}. \tag{25}$$

This value characterizes the number of years in which the gap between the actual GDP p.w. in region *i* ($y_i(t)$) and GDP p.w. in the steady-state (y_i^*) reduces by half.

3. The methods of calibration of the models

3.1.

The elasticities of GDP with respect to the physical capital in the Solow-Swan model were computed from the necessity conditions of maximizing the profit by producers:

$$\begin{aligned}&\Pi_i\left(K_i(t), L_i(t)\right) = \left\{A_i K_i^{\alpha_i}(t) L_i^{1-\alpha_i}(t) - r_i K_i(t) - w_i L_i(t)\right\} \to \max,\\ &K_i(t),\, L_i(t) \geq 0\end{aligned} \tag{26}$$

thus:
$$(1-\alpha_i) = \frac{w_i}{A_i K_i^{\alpha_i}(t) L_i^{-\alpha_i}(t)} = \frac{w_i L_i(t)}{A_i K_i^{\alpha_i}(t) L_i^{1-\alpha_i}(t)} = \frac{w_i L_i(t)}{Y_i(t)} \tag{27}$$

$$\alpha_i = 1 - \frac{w_i L_i(t)}{Y_i(t)} \tag{28}$$

where: w_i stands for average yearly wages in the region *i.*

3.2.

It was assumed in the Mankiw-Romer-Weil model that elasticity of human capital is equal to the elasticity of labor. The elasticities of physical capital were calculated as in the Solow-Swan model, while the elasticities of human capital were calculated according to formula:

$$\beta_i = \frac{1}{2}\frac{w_i L_i(t)}{Y_i(t)}. \tag{29}$$

3.3.

The values of total productivity factor A_i in the Cobb-Douglas production function were calculated to fit the initial GDP (given initial capital). Thus we have used the following equation:

$$A_i = \frac{y_i(0)}{k_i^{\alpha_i}(0)} \tag{30}$$

in the Solow-Swan model.

In the Mankiw-Romer-Weil model we have used equation:

$$A_i = \frac{y_i(0)}{k_i^{\alpha_i}(0)h_i^{\beta_i}(0)}. \tag{31}$$

3.4.

To get "true" trajectories of GDP *p.w.* for the Solow-Swan and Mankiw-Romer-Weil models we solved numerically differential equations (8) and (20) under initial year 1999 using Runge-Kutta method[3] implemented as an MATLAB function. Then we substituted computed capital trajectories as arguments into production functions and found the number of years (periods) needed to shrink the distance between initial GDP and steady state levels by factor 2, 4, … and so on. To find GDP trajectories and the corresponding half-convergence lengths (half-periods) in the linearized versions we proceeded analogously.

4. Empirical analysis in the neoclassical growth models for Polish regions

4.1. Parameters and results for the Solow-Swan model

In Table 1 there are parameters for the Solow-Swan model[4]. The most important for the value of GDP in steady-states are the parameters describing the elasticity

[3] See Burden R., Faires J., (1998),

[4] Abbreviations: POL – Poland, DOL – Dolnośląskie, KUJ – Kujawsko-Pomorskie, LUL – Lubelskie, LUS – Lubuskie, LOD – Łódzkie, MAL – Małopolskie, MAZ – Mazowieckie, OPL – Opolskie, PKR – Podkarpackie, PDL – Podlaskie, POM – Pomorskie, SLA – Śląskie, SWI – Świętokrzyskie, WRM – Warmińsko-Mazurskie, WIE – Wielkopolskie, ZAC – Zachodniopomorskie.

of production with respect to physical capital. As we can see the value of this parameter varies significantly among regions. The lowest values are in LUL (0.3167), PKR (0.3504) and in SWI (0.3551). The highest values are in ZAC (0.6039), LUS (0.5966) and DOL (0.5956). In the other regions the values of this parameter lie in the range from about 0,4 to about 0,55.

Parameter A_i of the production function is also known as total productivity factor. The greatest value of this parameter is in LUL (789.7) and SWI (559.3), the lowest in ZAC (44.2) and LUS (48.6). In general in the regions where the value of α_i is lower, the total productivity factor is higher.

Parameter s_{K_i} describes investment in the physical capital rate. It is the relation of total investment in physical capital in the region *i* to the GDP of the region *i*. As we can see, the investment rate was the greatest in MAZ (0.2874). The lowest values of this parameter were in WRM (0.1413), KUJ (0.1586) and in PDL (0.1645). In the Table 1 we marked out all the cases in which the investment rate was above the average. As one can see such a situation happened only in two regions: DOL and MAZ.

Table 1. The values of parameters in the Solow-Swan model

Parameters	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
A_i	90.5	51.9	79.2	**789.7**	48.6	**133.6**	**194.0**	**144.0**	72.2
α_i	0.5382	**0.5956**	**0.5545**	0.3167	**0.5966**	0.4992	0.4655	0.5079	**0.5454**
$\eta_i + \rho$[5]	0.0498	0.0484	**0.0502**	0.0483	**0.0507**	0.0453	**0.0535**	**0.0507**	0.0473
s_{K_i}	0.2048	**0.2215**	0.1586	0.1602	0.2016	0.1731	0.1859	**0.2874**	0.1985
Parameters	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
A_i	90.5	**585.6**	**340.7**	67.4	78.8	**559.3**	86.6	80.8	44.2
α_i	0.5382	0.3504	0.3999	**0.5651**	**0.5600**	0.3551	0.5340	**0.5541**	**0.6039**
$\eta_i + \rho$	0.0498	**0.0520**	0.0489	**0.0537**	0.0457	0.0484	**0.0521**	**0.0519**	**0.0508**
s_{K_i}	0.2048	0.1710	0.1645	0.1959	0.1892	0.1773	0.1413	0.2002	0.1629

A_i – total productivity factor, α_i – elasticity of production with respect to physical capital, $\eta_i + \rho$ – real depreciation rate, s_{K_i} – investment in physical capital rate.

Table 2 contains the actual values of GDP *p.w.* y_i^f and the values of these variables in the steady-states of Solow-Swan model y_i^{*SOL}. The GDP *p.w.* in the steady-states is the highest in DOL (163 296) and MAZ (145 786). The lowest values of GDP *p.w.* in the steady-states are in LUL (30 329), PKR (34 617) and PDL (37 263). The reason is that in these regions the investment rates and elasticities of production with respect to physical capital are low.

[5] We take ρ = 0,05 as the ratio of depreciation of physical (or human) capital.

In the Table 2 we have marked out the cases in which the values of GDP *p.w.* are higher than the average for Poland. As one can see in the steady-state the richest regions will remain rich. In seven regions (DOL, LUS, MAZ, POM, SLA, WIE, ZAC) the GDP *p.w.* is higher than the average. In the steady-state in five of them (DOL, LUS, MAZ, SLA, WIE) the GDP *p.w.* will be higher than the average value for Poland.

Table 2. The actual values and steady-state values of GDP *p.w.* in the Solow-Swan model (in PLN 1999)

Variables	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
y_i^f	43 159	**49 772**	42 084	27 326	**45 608**	38 340	35 984	**55 938**	40 646
y_i^{*SOL}	89 675	**163 296**	76 489	30 329	**116 849**	66 943	56 478	**145 786**	68 428
Variables	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
y_i^f	43 159	27 908	30 606	**49 638**	**51 376**	29 760	40 180	**43 973**	**50 680**
y_i^{*SOL}	89 675	34 617	37 263	86 050	**124 476**	37 219	45 117	**101 344**	84 425

y_i^f – actual GDP *p.w.*, y_i^{*SOL} – GDP *p.w.* in steady-state in the Solow-Swan model.

To see how much different parameters influence the values of GDP p.w. in steady-states, we have computed the parameters elasticities of GDP *p.w.* in steady-states. The results are given in Table 3.

As we can see, the greatest influence on GDP *p.w.* in steady-state has the parameter α_i – the elasticity of production with respect to physical capital. The elasticity of this parameter is several times greater than the elasticities of other parameters. For example, if the value of α_i in the region ZAC increases by 1% then, according to the Table 3, the GDP *p.w.* in steady-state increases by about 19%. The changes of total productivity factor, depreciation rate and investment rate cause much smaller changes in steady-state values. For example, the growth of total productivity factor by 1% in ZAC changes the value of GDP *p.w.* in the steady-state only by 2.52%.

Table 4 contains the relations of capital (GDP) p.w. in the regions to the capital (GDP) p.w. in Poland. There are actual relations and the relations in the steady-states. As we can see, the relations in the steady-states for some regions change significantly. The regions that will lose their positions while converging to steady-states are: KUJ, LUL, LOD, MAL, OPL, PKR, POD, POM, SWI, WRM and ZAC. The other regions will improve their position as regards steady-states relations of GDP *p.w.* in these regions to the average GDP *p.w.* in Poland. The great winner is region DOL. The relation of GDP *p.w.* in this region to the GDP *p.w.* in Poland is 1.153,

Table 3. Elasticities of GDP p.w. in the steady-states with respect to parameters

Elasticities	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
$e_{A_i}^{y_i^*}$	2,165	**2,473**	**2,245**	1,464	**2,479**	1,997	1,871	2,032	**2,200**
$e_{\alpha_i}^{y_i^*}$	14,939	**19,917**	**15,427**	5,340	**19,297**	12,414	10,615	14,061	**15,076**
$e_{\eta_i+\rho}^{y_i^*}$	–1,165	**–1,473**	**–1,245**	–0,464	**–1,479**	–0,997	–0,871	–1,032	**–1,200**
$e_{s_{K_i}}^{y_i^*}$	1,165	**1,473**	**1,245**	0,464	**1,479**	0,997	0,871	1,032	**1,200**
Elasticities	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
$e_{A_i}^{y_i^*}$	2,165	1,539	1,666	**2,299**	**2,273**	1,551	2,146	**2,243**	**2,525**
$e_{\alpha_i}^{y_i^*}$	14,939	6,280	7,824	**16,443**	**16,738**	6,509	13,425	**16,001**	**19,072**
$e_{\eta_i+\rho}^{y_i^*}$	–1,165	–0,539	–0,666	**–1,299**	**–1,273**	–0,551	–1,146	**–1,243**	**–1,525**
$e_{s_{K_i}}^{y_i^*}$	1,165	0,539	0,666	**1,299**	**1,273**	0,551	1,146	**1,243**	**1,525**

$e_{A_i}^{y_i^*}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter A_i (total productivity factor), $e_{\alpha_i}^{y_i^*}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter (elasticity of production with respect to physical capital), $e_{\eta_i+\rho}^{y_i^*}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter (real depreciation rate), $e_{s_{K_i}}^{y_i^*}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter (investment rate).

while in the steady-state this relation will be 1.821, that is the GDP *p.w.* in this region will be almost twice as big as the average GDP *p.w.* in Poland.

As one can see (in Table 5) the values of beta-coefficients in the Polish regions are very similar. In almost each region the value of this parameter lies within the range from about 2% to about 3.4%. The highest values of beta-coefficient are in PKR (3.38%), LUL (3.30%), SWI (3.12%) and PDL (2.93%). In these regions the convergence toward the steady-states is most rapid. As one can notice these are the regions in which the GDP *p.w.* in steady-state is relatively low. On the other hand, the beta-coefficients are low in DOL (1.96%), SLA (2.01%) and ZAC (2.01%) – in the regions where the level of GDP *p.w.* in the steady-states is relatively high.

The value t_i^{SOL} is the time of half-convergence. It is the number of years in which the gap between the current value of GDP p.w. and the value in the steady-state reduces by half. Of course this period is shorter in the regions where the value of

Table 4. Relations of the GDP *p.w.* in regions to GDP *p.w.* in Poland: the actual values and the values in the steady-states

Relations	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
y_i^f / y^f	**1.153**	0.975	0.633	**1.057**	0.888	0.834	**1.296**	0.942
y_i^{*SOL} / y^{*SOL}	**1.821**	0.853	0.338	**1.303**	0.747	0.630	**1.626**	0.763
Relations	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
y_i^f / y^f	0.647	0.709	**1.150**	**1.190**	0.690	0.931	**1.019**	**1.174**
y_i^{*SOL} / y^{*SOL}	0.386	0.416	0.960	**1.388**	0.415	0.503	**1.130**	0.941

y_i^f / y^f – relation of actual GDP *p.w.* in the region *i* to the actual GDP *p.w.* in Poland, y_i^{*SOL} / y^{*SOL} – relation of GDP *p.w.* in the steady-state in the region *i* to GDP *p.w.* in the steady-state in Poland.

Table 5. The beta-coefficients (speed of convergence) and the times of half-convergence

Coefficients	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
β_i^{SOL}	0,0230	0,0196	0,0224	**0,0330**	0,0205	0,0227	**0,0286**	**0,0249**	0,0215
t_i^{SOL}	30,2	**35,4**	**31,0**	21,0	**33,9**	**30,6**	24,3	27,8	**32,2**
Coefficients	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
β_i^{SOL}	0,0230	**0,0338**	**0,0293**	**0,0234**	0,0201	**0,0312**	**0,0243**	**0,0231**	0,0201
t_i^{SOL}	30,2	20,5	23,6	29,7	**34,5**	22,2	28,5	29,9	**34,5**

β_i^{SOL} – beta-coefficient in the Solow-Swan model, t_i^{SOL} – the time of half-convergence in the Solow-Swan model.

beta-coefficient is higher. In all Polish regions the time of half-convergence is about 20–30 years.

An important question in the convergence literature is how fast is the convergence of growth paths to their steady-states. The speed of convergence is usually measured with the help of beta parameters and half-convergence times (see the previous Table). But the values of these halftimes are computed on the basis of a linearized around steady-state version of equation (10). We wanted to see if the "true" half-times i.e. those which were computed on the basis of equation (10) are close to the theoretical ones originating from beta-values (see equation 15).

Table 6. Number of years needed to decrease the distance of GDP p.w. from steady-state level by half in the Solow-Swan model

Number of years	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
T(0)–T(d/2)	32	41	33	21	38	31	24	29	34
T(d/2) –T(d/4)	30	37	31	20	35	31	24	27	32
T(d/4)–T(d/8)	31	36	32	21	35	30	24	28	33
T(d/8)–T(d/16)	30	36	31	21	34	31	24	28	32
T(d/16)–T(d/32)	30	35	31	21	34	30	25	28	32
T(d/32)–T(d/64)	31	36	31	21	34	31	24	28	33
T(d/64)–T(d/128)	30	35	31	21	34	30	24	28	32
t_i^{SOL}	30	35	31	21	34	31	24	28	32
Number of years	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
T(0)–T(d/2)	32	20	23	32	37	21	29	32	37
T(d/2) –T(d/4)	30	20	24	30	36	22	29	31	36
T(d/4)–T(d/8)	31	20	23	30	35	22	28	30	35
T(d/8)–T(d/16)	30	20	24	30	34	22	29	30	34
T(d/16)–T(d/32)	30	21	23	29	35	22	29	30	35
T(d/32)–T(d/64)	31	20	24	30	34	22	28	30	34
T(d/64)–T(d/128)	30	21	23	30	35	22	29	30	35
t_i^{SOL}	30	21	24	30	34	22	29	30	34

T(d/x) denotes the number of years (periods) needed to reduce the distance of current GDP *p.w.* to its steady state not greater than d/x, where d is the distance at t = 0 and x = 2,4,…

It can be seen that the halftime values for the Solow-Swan model are not significantly different from their estimations based on the linearized model. One does not make a serious abuse while using beta based halftimes to estimate the speed of convergence of economies towards their steady state.

4.2. Parameters and results for the Mankiw-Romer-Weil model

Table 7 contains the values of parameters for the model with human capital. The parameters α_i, $\eta_i + \rho$, and s_{K_i} have the same values as in the model without human capital. The values of total productivity factor A_i are now different. In the model with human capital the values of this parameter are higher than in the model without human capital. For example, while in the Solow-Swan model the total productivity factor for the whole of Poland was 90.5, now it is 132. The reason for this difference lies in the way in which the parameters were calibrated.

While the values of total productivity factor change, the relations between the regions remain the same. As in the Solow-Swan model the highest values of this

Table 7. The values of parameters in the Mankiw-Romer-Weil model

Parameters	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
A_i	132.0	74.8	119.8	**1421.8**	72.2	**181.7**	**337.3**	**217.7**	**141.5**
α_i	0.5132	**0.5684**	**0.5250**	0.2773	**0.5683**	0.4836	0.4267	0.4800	0.4960
β_i	0.2434	0.2158	0.2375	**0.3613**	0.2158	**0.2582**	**0.2866**	**0.2600**	**0.2520**
$\eta_i + \rho$	0.0498	0.0484	**0.0502**	0.0483	**0.0507**	0.0453	**0.0535**	**0.0507**	0.0473
s_{K_i}	0.2048	**0.2215**	0.1586	0.1602	0.2016	0.1731	0.1859	**0.2874**	0.1985
s_{H_i}	0.3956	0.3618	**0.4399**	**0.4816**	0.3833	**0.4145**	**0.4519**	0.2965	**0.4230**
Parameters	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
A_i	132.0	**1065.7**	**631.3**	70.8	108.3	**901.5**	117.3	122.4	58.2
α_i	0.5132	0.3104	0.3550	**0.5730**	**0.5372**	0.3278	**0.5151**	**0.5249**	**0.5874**
β_i	0.2434	**0.3448**	**0.3225**	0.2135	0.2314	**0.3361**	0.2424	0.2375	0.2063
$\eta_i + \rho$	0.0498	**0.0520**	0.0489	**0.0537**	0.0457	0.0484	**0.0521**	**0.0519**	**0.0508**
s_{K_i}	0.2048	0.1710	0.1645	0.1959	0.1892	0.1773	0.1413	0.2002	0.1629
s_{H_i}	0.3956	**0.4638**	**0.4530**	**0.4260**	**0.4132**	**0.4393**	**0.4366**	**0.4362**	0.3768

A_i – total productivity factor, α_i – elasticity of production with respect to physical capital, β_i – elasticity of production with respect to human capital, $\eta_i + \rho$ – real depreciation rate, s_{K_i} – investment in physical capital rate, s_{H_i} – investment in human capital rate (in the region i or in Poland).

parameter are in LUL (1421.8), PKR (1065.7) and SWI (910.5). The lowest values are in ZAC (58.2), LUS (72.2) and DOL (74.8).

The parameter β_i describes the elasticity of production with respect to human capital. Its value has a very great influence on the value in the steady-state. The greater β_i, the higher is the value GDP *p.w.* in the steady-state. Because of the calibration method the parameter β_i has higher values in the regions where the parameter α_i has lower values. As one can see, the highest values of β_i are in LUL (0.3613), PKR (0.3448), PDL (0.3225) and SWI (0.3361), while the lowest values are in ZAC (0.2063), POM (0.2135), DOL (0.2158) and LUS (0.2158).

The rate of investment in human capital s_{H_i} was estimated as a relation of the local government spending on education to the total value of local government spendings. As one can see in many regions the values of this parameter were higher than the average for Poland. These regions are marked out in the Table 7. The highest values of s_{H_i} are in LUL (0.481) and PKR (0.4638), while the lowest value is in MAZ (0.2965).

Table 8 contains the values of the GDP *p.w.* in the steady-states $\overset{*}{y}{}^{MRW}$. For comparison we also put there the actual values of these variables y_i^f. What makes the greatest impression in this Table is the level of variables in the steady-states. They are a million times greater than the actual values. It seems, at the first sight, that this result does not make sense. We will try to argue here that the results make sense, but that the things that really matter are the relations between values in the steady-states in different regions, not their actual values.

Table 8. The actual values of GDP *p.w.* and the steady-states values of these variables in the Mankiw-Romer-Weil model (in PLN 1999)

Variables	POL	DOL	KUJ	LUL	LUS
y_i^f	35 604	**41 059**	34 717	22 542	**37 624**
y^{*}_{MRW}	80 904 117 434	**197 217 897 247**	62 972 093 644	13 278 276 642	**116 664 195 297**
Variables	POL	LOD	MAL	MAZ	OPL
y_i^f	35 604	31 628	29 685	**46 146**	33 531
y^{*}_{MRW}	80 904 117 434	63 639 859 303	35 676 657 826	**141 187 187 658**	51 781 780 599
Variables	POL	PKR	PDL	POM	SLA
y_i^f	35 604	23 023	25 248	**40 949**	**42 382**
y^{*}_{MRW}	80 904 117 434	15 742 845 609	16 975 693 740	**118 099 951 129**	**152 275 196 068**
Variables	POL	SWI	WRM	WIE	ZAC
y_i^f	35 604	24 551	33 147	**36 276**	**41 808**
y^{*}_{MRW}	80 904 117 434	19 966 546 988	23 878 466 404	**102 085 188 352**	**74 116 605 551**

y_i^f – actual GDP *p.w.*, y^{*}_{MRW} – GDP *p.w.* in steady-state in Mankiw-Romer-Weil.

In the computation we have taken as the human capital the number of workers who graduated from secondary school. The reason was that to calibrate the model, we have to find an empirical equivalent of the human capital. It seems that the number of educated workers fits here the best. But in fact we should rather assume that the „real'' human capital is proportional to the number of educated workers. That is, every educated worker has some amount C of human capital. The „real'' human capital is thus

$$\tilde{H} = CH, \tag{32}$$

where H is the number of workers who graduated from secondary school. If we express it in per worker terms, we can see that the „real'' human capital p.w. is proportional to h:

$$\tilde{h} = Ch. \tag{33}$$

The values of variables in the steady-states change according to the following formula:

$$\overset{*}{\tilde{k}}_{\mathrm{i}}{}^{\mathrm{MRW}} = \frac{\overset{*}{k}_{\mathrm{i}}{}^{\mathrm{MRW}}}{C^{\frac{1}{1-\alpha-\beta}}},\ \overset{*}{\tilde{h}}_{\mathrm{i}}{}^{\mathrm{MRW}} = \frac{\overset{*}{h}_{\mathrm{i}}{}^{\mathrm{MRW}}}{C^{\frac{1}{1-\alpha-\beta}}},\ \overset{*}{\tilde{y}}_{i}{}^{\mathrm{MRW}} = \frac{\overset{*}{y}_{i}{}^{\mathrm{MRW}}}{C^{\frac{1}{1-\alpha-\beta}}}. \tag{34}$$

That is, the "real" steady-state values are proportional to the ones given in Table 8. If we knew the value of C, we could easily compute the "real" steady-states in

Table 9. Relations of the GDP *p.w.* in regions to GDP *p.w.* in Poland: the actual values and the values in the steady-states

Relations	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
y_i^f / y^f	**1.153**	0.975	0.633	**1.057**	0.888	0.834	**1.296**	0.942
$\overset{*}{y}_i{}^{MRW} / \overset{*}{y}{}^{MRW}$	**2.438**	0.778	0.164	**1.442**	0.787	0.441	**1.745**	0.640
Relations	**PKR**	**PDL**	**POM**	**SLA**	**SWI**	**WRM**	**WIE**	**ZAC**
y_i^f / y^f	0.647	0.709	**1.150**	**1.190**	0.690	0.931	**1.019**	**1.174**
$\overset{*}{y}_i{}^{MRW} / \overset{*}{y}{}^{MRW}$	0.195	0.210	**1.460**	**1.882**	0.247	0.295	**1.262**	0.916

y_i^f / y^f – relation of actual GDP *p.w.* in the region i to the actual GDP *p.w.* in Poland, $\overset{*}{y}_i{}^{MRW} / \overset{*}{y}{}^{MRW}$ – relation of GDP *p.w.* in the steady-state in the region *i* to GDP *p.w.* in the steady-state in Poland.

Table 10. Parameters of elasticities of GDP *p.w.* in steady-state in the Mankiw-Romer-Weil model

Elasticities	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
$e_{A_i}^{\overset{*}{y_i}}$	4,109	**4,634**	**4,211**	2,767	**4,633**	3,873	3,489	3,846	3,969
$e_{\alpha_i}^{\overset{*}{y_i}}$	55,943	**72,502**	**57,519**	18,809	**70,726**	49,113	38,031	50,594	51,390
$e_{\beta_i}^{\overset{*}{y_i}}$	27,710	**28,019**	**27,037**	25,608	**27,505**	27,090	26,432	27,440	26,861
$e_{\eta_i+\rho}^{\overset{*}{y_i}}$	–3,109	**–3,634**	**–3,211**	–1,767	**–3,633**	–2,873	–2,489	–2,846	–2,969
$e_{s_{K_i}}^{\overset{*}{y_i}}$	2,109	**2,634**	**2,211**	0,767	**2,633**	1,873	1,489	1,846	1,969
$e_{s_{H_i}}^{\overset{*}{y_i}}$	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
Elasticities	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
$e_{A_i}^{\overset{*}{y_i}}$	4,109	2,900	3,101	**4,684**	**4,322**	2,975	**4,125**	**4,210**	**4,848**
$e_{\alpha_i}^{\overset{*}{y_i}}$	55,943	22,211	27,266	**71,888**	**63,083**	24,400	**52,888**	**58,997**	**74,594**
$e_{\beta_i}^{\overset{*}{y_i}}$	27,710	25,668	25,781	**27,566**	**27,952**	25,923	**26,022**	**27,478**	**27,034**
$e_{\eta_i+\rho}^{\overset{*}{y_i}}$	–3,109	–1,900	–2,101	**–3,684**	**–3,322**	–1,975	**–3,125**	**–3,210**	**–3,848**
$e_{s_{K_i}}^{\overset{*}{y_i}}$	2,109	0,900	1,101	**2,684**	**2,322**	0,975	**2,125**	**2,210**	**2,848**
$e_{s_{H_i}}^{\overset{*}{y_i}}$	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

$e_{A_i}^{\overset{*}{y_i}}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter A_i (total productivity factor), $e_{\alpha_i}^{\overset{*}{y_i}}$ –elsaticity of GDP *p.w.* in the steady-state with respect to parameter β_i (elasticity of production with respect to physical capital), $e_{\beta_i}^{\overset{*}{y_i}}$ – elsaticity of GDP *p.w.* in the steady-state with respect to parameter $\eta_i + \rho$ (elasticity of production with respect to human capital), $e_{\eta_i+\rho}^{\overset{*}{y_i}}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter (real depreciation rate), $e_{s_{K_i}}^{\overset{*}{y_i}}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter s_{K_i} (investment in physical capital rate), $e_{s_{H_i}}^{\overset{*}{y_i}}$ – elasticity of GDP *p.w.* in the steady-state with respect to parameter s_{H_i} (investment in human capital rate).

the model. However we do not know this value. We can only compute the relations between variables in the steady-states because these relations do not change.

Table 9 contains the relations of GDP *p.w.* in regions to the GDP *p.w.* in Poland. There are actual relations and relations in the steady-states. As in the model without human capital a great winner is region DOL. Now the relation of GDP *p.w.*

in this region to the GDP *p.w.* in Poland is 1.153, while in the steady-state this relation will be almost 2.5.

To see how much different parameters influence the values of capital and GDP *p.w.* in the steady-states, we have computed the parameters elasticities of GDP *p.w.* in the steady-states. The results are given in Table 10. As we can see, the greatest influence on GDP *p.w.* in the steady-states have the parameters α_i and β_i – the elasticities of production with respect to physical and human capital. The elasticities of these parameters are several times greater than the elasticities of the other parameters. One can also notice that the elasticities of α_i are much higher than in the Solow-Swan model. The elasticities of α_i are usually higher than the elasticities β_i of with the exception for LUL, PKR and SWI. The elasticities of investment in human capital rate $e^{y_i^*}_{s_{H_i}}$ are always equal to 1.

Table 11 contains the beta-coefficients and the times of half-convergence. The speed of convergence in the model with human capital is lower than in the model without it – the beta-coefficients are now lower and the periods of half-convergence are longer. It turns out that it takes from about 40 years (LUL) to over 66 years (ZAC) to reduce the gap to the steady-states by half.

Table 11. The beta-coefficients (speed of convergence) and the times of half-convergence

Coefficients	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
t_i^{MRW}	0,0121	0,0104	0,0119	**0,0175**	0,0110	0,0117	**0,0153**	**0,0132**	0,0119
β_i^{MRW}	57,2	**66,3**	**58,2**	39,7	**63,3**	**59,3**	45,2	52,6	**58,2**
Coefficients	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
t_i^{MRW}	0,0121	**0,0179**	**0,0158**	0,0115	0,0106	**0,0163**	**0,0126**	**0,0123**	0,0105
β_i^{MRW}	57,2	38,7	44,0	**60,4**	**65,6**	42,6	54,9	56,2	**66,2**

β_i^{MRW} – beta-coefficient in the Mankiw-Romer-Weil model, t_i^{MRW} – the time of half-convergence in the Mankiw-Romer-Weil model.

Table 12 lets us compare the "true" speed of convergence with its estimate contained in Table 11.

T(d/x) denotes the number of years (periods) needed to reduce the distance of current GDP *p.w.* to its steady-state not greater than d/x, where d is distance at $t = 0$ and $x = 2, 4, \dots$

It follows that the "true" values of half-periods are around twice as the theoretical ones at the beginning. When one takes into account that the theoretical half-periods are rather long (from 39 to 66 years) it seems that the difference is signifi-

Table 12. Number of years needed to decrease the distance of GDP *p.w.* from steady-state level by half in the Mankiw-Romer-Weil model

Distances	POL	DOL	KUJ	LUL	LUS	LOD	MAL	MAZ	OPL
T(d/2)	132	165	136	65	157	131	92	116	130
T(d/2) -T(d/4)	67	79	68	44	76	69	52	61	69
T(d/4)-T(d/8)	61	72	63	41	68	63	48	56	62
T(d/8)-T(d/16)	60	68	60	41	65	62	47	55	60
T(d/16)-T(d/32)	58	68	59	40	65	60	46	53	59
T(d/32)-T(d/64)	57	-	59	40	64	60	45	53	59
T(d/64)-T(d/128)	58	-	-	39	-	-	46	53	58
t_i^{MRW}	57	66	58	40	63	59	45	53	58
Distances	POL	PKR	PDL	POM	SLA	SWI	WRM	WIE	ZAC
T(d/2)	132	66	80	151	156	75	126	132	168
T(d/2) -T(d/4)	67	44	50	72	78	48	64	66	80
T(d/4)-T(d/8)	61	40	47	65	70	45	59	60	71
T(d/8)-T(d/16)	60	40	45	63	68	44	57	58	69
T(d/16)-T(d/32)	58	39	44	61	67	43	56	57	67
T(d/32)-T(d/64)	57	39	44	61	-	43	55	57	-
T(d/64)-T(d/128)	58	39	45	-	-	42	55	57	-
t_i^{MRW}	57	39	44	60	66	43	55	56	66

cant. Thus one should be cautious when transposing some results from a linearized version of the Mankiw-Romer-Weil model into the initial one in the MRW model, which is opposite to the situation in the Solow-Swan model.

5. Conclusion

Figure 1 contains the final results of the performed experiment. It presents distribution of GDP *p.w.* in the Polish regions – the actual one and in the steady-states.

Taking into consideration all simplifications in the assumptions of the models and in the computation procedure, we can conclude that the long-run distributions of GDP *p.w.*, obtained on the basis of Solow-Swan and Mankiw-Romer-Weil models, show significant inequalities between the regions.

The richest regions in 1999 like DOL, MAZ, SLA will improve their positions, while the poorest regions like LUL, PDK, SWI will lose their positions as compared to the average level in Poland. There is also a group of regions with the wealth close to the Polish average, like WIE, LUS, POM. These regions will slightly improve their position but in the long run they will still be close to the Polish average.

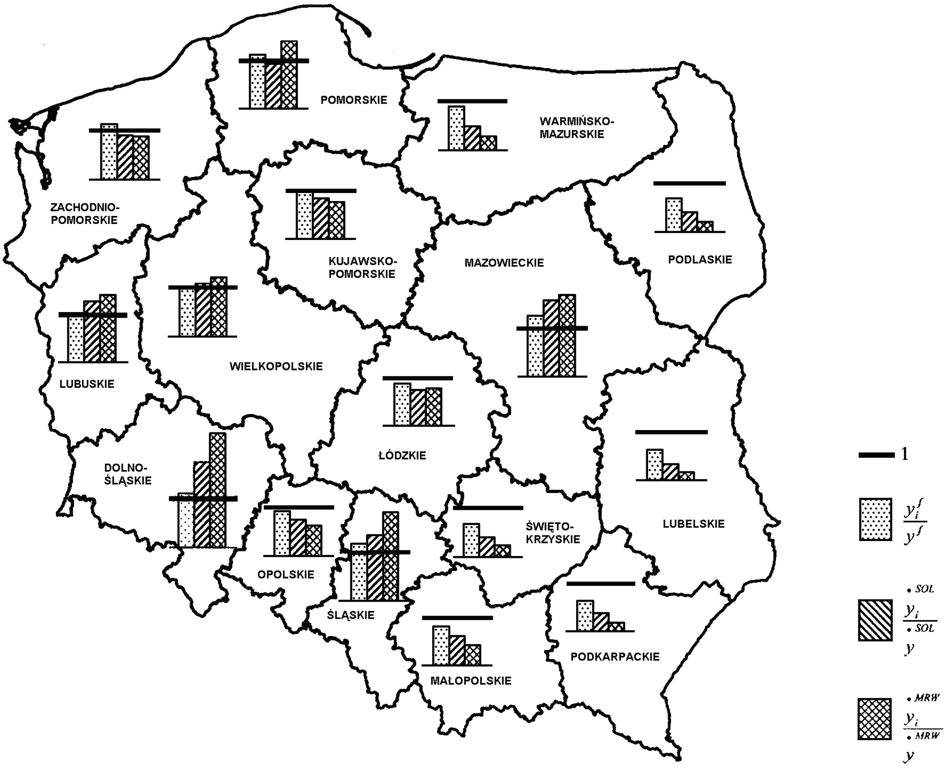

Figure 1. Relations of the GDP *p.w.* in regions to GDP *p.w.* in Poland: the real values and the values in the steady-states

This tendency is more visible in the results obtained from the Mankiw-Romer-Weil model than from the Solow-Swan model.

The application of the neoclassical growth models of Solow-Swan and Mankiw-Romer-Weil in the analysis of regional inequalities in Poland is a starting point for the discussion on the usefulness of the neoclassical growth models in the research into long-term inequalities in a chosen country.

The gist of the undertaken experiment was an analysis of hypothetic GDP *p.w.* paths that stem from the values of calibrated parameters and the comparison of GDP *p.w.* in the steady -states among the Polish regions in 1999. Simplicity of the applied models, method of parameters calibration and the convergence speed towards the steady-states measure broaden our understanding of the real and hypothetical regional inequalities in Poland.

The main conclusion is that inequalities will grow – the "rich" regions will become richer and the "poor" ones, relative position will worsen – even though their absolute wealth level will not change essentially. In the central and east part of

Poland – besides MAZ region – the models predict a radical wealth decrease in comparison to the average value of wealth measured as the average GDP *p.w*. in Poland.

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