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Wiesław ŁUCZYŃSKI
Poznań University of Economics

The long memory dynamics of the market quotations of selected Stock Companies and Warsaw Stock Index[1]

Abstract. The analysis of the results obtained allows to establish the short memory of the stationary time series deprived of the stochastic trend, the permanent memory of the non stationary data of the Hodrick-Prescott trend and the declining memory of the non stationary time series of the quotation dynamics of the stock companies. As results from the research, the spectral analysis of the stock quotations does not reveal the fluctuations in the business conditions. The fluctuations with the period of about 34, 20 and 14 sessions with the quotations of particular companies and WIG correspond to the periods 8-10, 5-6 and 3-4 weeks.

Keywords: stationarity, spectral analysis, R/S analysis, fractional integration, wavelet analysis.

JEL codes: C22, G12.

From the empirical point of view the long memory is associated with persistence of the observed phenomena. This phenomenon was initially noted down not in economics but in such domains as hydrology, geophysics, climatology, and in other natural sciences[2]. The weakening self correlation of the time series has attracted the attention of the researchers for a long time. The works of J. Beran, R.T. Baillie, P. Doukham, G. Oppenheim, M.S. Taqqu and P.M. Robinson[3] include the compre-

[1] I am very much indebted to Aurelia Łuczyński, who helped me to solve technical problems related to my work.

[2] The long memory researches were initiated about the year 1907 by H. E. Hurst, a hydrologist [Hurst, 1951]. H.E.Hurst discovered that the most natural systems do not meet the conditions connected with the at random missing i.e. the trend associated with the information disturbance. The reaction of these systems to the new information is differentiated, which is connected with its different absorption (dependings on whether the new information confirms the earlier observations or not e.g. tendency – trend). E. E. Peters developed the results obtained by Hurst and added the time lines associated with the economic processes, particularly with the capital markets [Peters,1997, s. 64-119]; and [Mandelbrot, Wallis, 1968], [Mandelbrot, 1972], [McLeod, Hipel, 1978].

[3] See [Beran, 1994], [Baillie, 1996], [Theory and applications of long-range dependence, 2003], [Robinson, 1994, 2003].

hensive review of the long memory processes and the review of the fractional integration. However, the recent scientific research refers to nonstationarity, long memory and nonlinearity time series[4]. The starting point of the research on the fractionally integrated processes is the fact that very often the economic and financial time series are not integrated processes, either of order zero or of order d = 1: I(0), I(1). Autocorrelation appears in these series after a long time. However, in a given long period the autocorrelation disappears constantly (in literature this phenomenon is known as the "hyperbolic decay". [Banerjee, Urga, 2004, p. 15]. The typical feature of the long memory processes is "overdifferencing" after obtaining the first time series differences containing the observations of these processes. In such conditions the integrated process of order '*d*' is defined as fractional or partial: d is the non integer number and it takes the values from the range $(-\infty, 0)$ for the memory-less or the weak memory processes; $(0, \frac{1}{2})$ for the stationary processes with long memory and from the range $(\frac{1}{2}, +\infty)$ for nonstationary processes with long memory.

The processes with long memory can be presented in the linear- time dimension or in the frequency dimension. In the linear-time dimension the long memory is presented as a hyperbolic decay of the autocorrelation function mentioned above. As the autocorrelation function covers the long time motions (lags) illustrating (on the graphs) the long memory of the processes with not a big number of observations is impossible. However, in the frequency dimension, the same information presented in the spectral form covers all the fluctuations within the range $(0, \pi)$. Therefore, using the spectral analysis allows us to illustrate on the graph the long memory processes, independently of how long period the statistic data cover. The definitions of the long memory process are differentiated depending on the time dimension assumed in the researches.

In the linear-time dimension, the stationary, discrete time series includes the long dependences (or the long memory), if its correlation function ρ_j for the delay (*lag*) *j*, satisfies the condition:

$$\lim_{j\to\infty} \frac{\rho_j}{c_\rho j^{-\alpha}} = 1, \tag{1}$$

for $0 < c_\rho < \infty$ and $0 < \alpha < 1$.

The definition given implies that the dependence between two optional observations gradually disappears simultaneously with extending the lag to infinity. And a more general definition suggested by A.I.McLeod and K.W.Hipel [McLeod, Hipel, 1978]:

[4] See Journal of Econometrics by J. Davidson and T.T. Terasvirt wholly dedicated to long memory and non linear time serious [Long memory and nonlinear time series, 2002].

$$\lim_{j\to\infty}\sum_{j=-n}^{n}\left|\rho_j\right|=\infty, \tag{2}$$

where n is the number of observations.

Both definitions (1) and (2) refer to the nonstationary processes. However their autocorrelation functions cannot decline too rapidly.

Another definition of the long memory is associated with the spectral density function. The process of the spectral density *f* is endued with the long memory if

$$\lim_{\lambda\to\infty}\frac{f(\lambda)}{c_f\,|\lambda|^{-\beta}}=1, \tag{3}$$

for a stable $0 < c_\rho < \infty$ and $0 < \beta < 1$.

This means that the value of the spectral density function for the zero frequency is infinite. Equation (3) is not identical to equation (2), but they are inter-connected [Beran, 1994]. If $\frac{1}{2} < H < 1$ then $\alpha = 2 - 2H$, and $\beta = 2H - 1$. H is defined as the Hurst coefficient or indicator. It was defined by H.E. Hurst [Hurst, 1951] and represents the classical parameter which allows us to establish the presence of the long memory in the processes being searched. The long memory of the process occurs if the Hurst coefficient $\frac{1}{2} < H < 1$ [Yong,1974], [Robinson, 1995]. Whereas for $-1 < \beta < 0$ ($0 < H < \frac{1}{2}$) the process is denoted by the 'minus memory' or the anti persistence (if the changes of the time series direction are more frequent than in the random series[5]).

In the analysis of the dynamics of the economic processes, when considering their numerous aspects, different tools should be applied. To such tools belong, among others, the spectral analysis, the rescaled range analysis and the wavelets analysis. Each of them treats to some extent differently the data from the observation of the researched processes. To obtain the time series with defined features they are subject to the initial procedure, the so-called prewhitening procedure. The most demanding in this respect is the spectral analysis. Moreover, reliability of the results depends directly proportionally on the dimension of the observation series and on the length of the period which they cover. It is obvious that the change of the conditions in which the researched processes are running, put under the question mark homogeneity of the collected observation series. The methods of their collecting and the statistical processing also change. As opposed to the uniform mathematical objects or (although inconsiderably) – physical objects, the economic systems are characterized by irreversibility of the time series. These systems assume the forms changeable in time. We should not forget it when we interpret the results of

[5] If in the previous period the system assumed high values, it is presumable that in the next period the turn will come and the system will accept the low values and vice versa. Such a system is defined as a pink noise or a noise 1/f [Peters, 1997, s. 242].

research into the dynamics of economic processes and when we attempt to predict their behaviour in the future.

The spectral analysis was carried out for the time series describing the quotation dynamics of the following companies Kable, Krosno,Próchnik and the WIG aggregation The stationarity of the researched time series was obtained by removing the stochastic Hodrick-Prescot trend from the data. The quotation data enfolded the period from 16 April 1991 till 27 March 2006. The dynamics indexes were obtained by treating the quotation level from the beginning of the period as the basis. Subsequently, the type of stationary series was established, by using the (ADF), Augmented Dickey-Fuller Test and the test by Kwiatkowski, Philips, Schmidt and Shin (KPSS).

At the initial stage of the development the Warsaw Stock Exchange operated once, twice or three times a week in the uniform system of the day course (the so-called fixing). From 3rd October 1994 the stock sessions started to take place every weekdays, and as before in the fixing system. This is, among others, the reason of such a differentiated number of observations in the time series quotations of Kable (2617 data), Krosno (2740 data), Próchnik (2618 data) and WIG (3289 data). Such big differentiation of the frequency of the quotations can have an influence on differentiation of delays in spreading the information signals and what is more, periodicity of the autocorrelation associations and differentiation of the time series variability in the researched period. However, the differentiation given should not have a bearing on the cyclical structure of the long term cyclical fluctuations (associated with the long memory processes). The most important here is not the frequency of sampling of the survey process but the length of the time period in which it runs (in this case almost 15 years). To obtain reliable results of the harmonic analysis of the time series, the research period should not be shorter than 10 multiplied lengths of the longest fluctuation period. So, for example, to verify the hypothesis about the 3 year fluctuations in the researched time series, such a series should not be shorter than 30 years. However, the frequency of measuring (sampling) of the researched process is less important here. The rising frequency of sampling results in increased disturbances of the information signals, and consequently, in more complex statistical processing (the so-called prewhitening).

The Augmented Dickey-Fuller Test is used to evaluate occurrence of nonstationarity in a variance of the researched process. Three types of regression models are used: the model without the intercept and the trend (4), the model with the intercept (5) and the model with the intercept and the trend (6).

$$\Delta Y_t = gY_{t-1} + u_t \tag{4}$$

$$\Delta Y_t = \alpha + gY_{t-1} + u_t \tag{5}$$

$$\Delta Y_t = \alpha + bT + gY_{t-1} + u_t. \tag{6}$$

The zero hypothesis H_0 says that, when g=0, i.e. the researched series (process) is nonstationary (the unit root exists, the accumulation or integration of the process occurs). However, the alternative hypothesis H_1 says that the series (process) is stationary, i.e. g<0 (the unit root does not exist, the accumulation or integration of the process do not occur). In our case models (5) and (6) are tested. The aim is to establish the type of stationarity of the researched series. Depending on whether the series are difference stationary or trend stationary, different procedures of removing non-stationarity are used [Gajda, 2004, pp. 149-158; Kufel, 2004, pp. 68-70; Gruszczyński, Kluza, Winek, 2003, pp. 159-164]. Nonstationarity can be brought into the process by the deterministic trend and/or by the stochastic trend. The difference stationary processes include the deterministic trend or the stochastic trend, whereas the trend stationary processes include exclusively the deterministic trend. The obtained results of testing the time series prove that we deal with the difference stationary phenomenon.

The results of the unit root test of the quotations dynamics of Kable, Krosno, Próchnik and WIG are as follows[6]:

```
     KBL_dyn;ADF tests (T=2617, Constant;5%=-2.86 1%=-3.44)
   D-lag        t-adf      beta Y_1       sigma         AIC
     0         -2.386      0.99585        18.82         5.870
  KBL_dyn:ADF tests (T=2617, Constant+Trend;5%=-3.41 1%=-3.97)
   D-lag        t-adf      beta Y_1       sigma         AIC
     0         -2.498      0.99560        18.82         5.871
      KR_dyn:ADF tests (T=2740,Constant;5%=-2.86 1%=-3.44)
   D-lag        t-adf      beta Y_1       sigma         AIC
     0          -1.99      0.99710        41.84         7.469
 KR_dyn:ADF tests (T=2740, Constant + Trend; 5%=-3.41 1%=-3.97)
   D-lag        t-adf      beta Y 1       sigma         AIC
     0         -1.727      0.99716        41.85         7.469
     PR_dyn:ADF tests (T=2618, Constant;5%=-2.86 1%=-3.44)
   D-lag        t-adf      beta Y 1       sigma         AIC
     0         -1.814      0.99744        21.65         6,151
  PR_dyn:ADF tests (T=2618, Constant+Trend; 5%=-3.41 1%=-3.97)
   D-lag        t-adf      beta Y 1       sigma         AIC
     0         -2.541      0.99560        21.64         6.151
    WIG_dyn: ADF tests (T=3289, Constant; 5%=-2.86 1%=-3.44)
   D-lag        t-adf      beta Y_1       sigma         AIC
     0         0.5439      1.0003         27.37         6.620
 WIG_dyn: ADF tests (T=3289, Constant+Trend; 5%=3.41 1%=-3.97)
   D-lag        t-adf      beta Y_1       sigma         AIC
     0        -0.5890      0.99934        7.37          6.620
```

[6] The testing was carried out with the help of module PcGive 10.0 – GiveWin 2 Educational – Single-User. Identical results were obtained in the GRETL programme (v. 1.5.1.) and EasyReg International (March, 12 2006).

The values of t-adf statistics do not allow us to reject the zero hypothesis about nonstationarity of the quotation dynamics series of the Stock Companies and WIG. However, it is more difficult to reject this hypothesis (except for the dynamics of Próchnik quotations) regarding the model which includes the intercept together with the trend, this would prove that the researched processes are stationary not towards the deterministic trend but towards difference (and the stochastic trend). Removing the undesired nonstationarity would in this case demand removing the stochastic trend. A similar conclusion was drawn from the research carried out by the author in 1998 [Łuczyński, 1998, p. 642], although the research included a comparatively short period of the quotations and more unstable (than now) conditions for the companies and the economy. Predicting the difference stationary series ((*y*) is more difficult because the consequences of the disturbances do not become weaker in time (as when predicting the trend stationary series i.e. stationary towards the deterministic trend) but they ”remain permanently “frozen into” the future values of the variable *y*” [Gajda, 2004, p. 156].

Testing the nonstationarity of the researched series was supplemented with the KPSS test. The Kwiatkowski and the others' test [Kwiatkowski, Phillips, Schmidt, Shin, 1992] develops the idea of testing the zero hypothesis that the series is stationary towards the deterministic trend or towards the intercept. When the calculated statistical values of KPSS exceed the critical values, there are no reasons to reject the zero hypothesis about the trend ‘stationarity (or about the trend-less stationarity, towards the intercept) of the researched series. In the opposite case the zero hypothesis can be rejected and replaced by the alternative hypothesis about nonstationarity of the series [Syczewska, 2005, p. 123]. The KPSS test allows (unlike the ADF test) to reject the zero hypothesis about the trend stationarity in the conditions when atypical behaviours of the researched processes occur [Otero, Smith, 2003, p. 2].

The KPSS test results obtained for the quotations dynamics of the stock companies and WIG do not allow to reject not only the zero hypothesis about their trend stationarity but also the zero hypothesis about stationarity without the deterministic trend (in relation to the intercept). At the same time, rejection of the second zero hypothesis proved to be more difficult. Therefore the KPSS test explicitly indicates the stationarity of the series towards the trend (including also the stochastic trend which indicates permanent and large anomalies). It would be an important recommendation for forecasting such processes. Simultaneously, the quality of forecasting could become radically improved after the localization of specific turning points where the change of the cyclical structure of the time series occurs.

```
The zero hypothesis: the stationary process; test KPSS for variab-
                     le KBL_dyn
                     (without trend)
                     Lag truncation = 0
                     The test statistics = 20,4545
                     10%       5%        2,5%      1%
Critical values:     0,347     0,463     0,574     0,739
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The zero hypothesis: the stationary process; the KPSS test for va-
                     riable KBL_dyn
                     (with trend)
                     Lag truncation = 0
                     The test statistics = 10,5926
                     10%       5%       2,5%     1%
Critical values:     0,119    0,146    ,176     0,216
The zero hypothesis: the stationary process; the KPSS test for va-
                     riable KR_dyn
                     (without trend)
                     Lag truncation = 0
                     The test statistics = 69,2624
                     10%       5%       2,5%     1%
Critical values:     0,347    ,463     0,574    0,739
The zero hypothesis: the stationary process; the KPSS test for va-
                     riable KR_dyn
                     (with trend)
                     Lag truncation = 0
                     The test statistics = 9,07095
                     10%       5%       2,5%     1%
Critical values:     0,119    0,146    0,176    0,216
The zero hypothesis: the stationary process: The KPSS test for va-
                     riable PR_dyn
                     (without trend)
                     Lag truncation = 0
                     The test statistics = 101,24
                     10%       5%       2,5%     1%
Critical values:     0,347    0,463    0,574    0.739
The zero hypothesis: the stationary process; the KPSS test for va-
                     riable PR_dyn
                     (with trend)
                     Lag truncation = 0
                     The test statistics = 7,24485
                     10%       5%       2,5%     1%
Critical values:     0,119    0,146    0,176    0,216
The zero hypothesis: the stationary process; The KPSS test for va-
                     riable WIG_dyn
                     (without the trend)
                     Lag truncation = 0
                     The test statistics = 207,51
                     10%       5%       2,5%     1%
Critical values:     0,347    0,463    0,574    0,739
The zero hypothesis: the stationary process; the KPSS test for va-
                     riable WIG_dyn
                     (with trend)
                     Lag truncation = 0
                     The test statistics = 24,0147
                     10%       5%       2,5%     1%
Critical value:      0,119    0,146    0,176    0,216
```

Both the ADF test and the KPSS test more or less explicitly indicate the stationarity towards the stochastic trend of the researched processes. Therefore the question arises whether removing the stochastic Hodrick – Prescott trend from the researched series of the quotation dynamics allows us to obtain an explicitly stationary (towards such a trend) time series. The results below give us a positive answer to the question posed. The data with the note _hp include the stochastic Hodrick – Prescott trend generated on the basis of the series of the quotation dynamics of the respective stock companies and WIG. The calculations were carried out using Add –In to the Excell Timeseries filters package by Kurt Annen[7]. The data with the note _cycle were obtained as a quotient of the original data and the trend data: y_cycle = y_dyn/y_hp.

-- -Give Win 2.30 session started at 21:30:01 on Monday 10 April 2006---
Ox version 3.30 (Windows) © J. A. Doornik, 1994 2003
Descriptive Statistics package version 1.0, object created on 10-04-2006

KBL_cycle: ADF tests (T=2619, Constant;5%=-2.86 1%=3.44)

D-lag	t-adf	beta Y_1	sigma	AIC
0	- 13.25**	0.87433	0.06025	-5.618

KBL_cycle: ADF tests (T =2619, Constant+Trend; 5%=3.41 1%=-3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-13.25**	0. 87425	0.6026	-5.617

KR_cycle: ADF tests (T=2740, Constant; 5%=-2.86 1%=3.44

D-lag	t-adf	beta Y_1	sigma	AIC
0	-8.289**	0.95127	0.05071	-5.963

KR_cycle: ADF tests (T=2740, Constant + Trend; 5%=-3.41 1%=3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-8.294**	0.95121	0.05071	-5.962

PR_cycle: ADF tests (T=2619, Constant; 5%=-2.86 1%=-3.44)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-11.24**	0.90839	0.06225	-5.553

PR_cycle: ADF tests (T=2619, Constant + Trend; 5%=-3.41 1%=-3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-11.25**	0.90809	0.06225	-5.552

WIG_cycle:ADF tests (T=3289, Constant; 5%= -2.86 1%=3.44)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-15.74**	0.86016	0.03620	-6.637

WIG_cycle: ADF tests (T=3289, Constant+Trend; 5%=-3.41 1%=3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-15.74**	0.86016	0.03620	-6.636

KBL_hp: ADF tests (T=2617, Constant; 5%=-2.86 1%=3.44)

D-lag	t-adf	beta Y_1	sigma	AIC

7 [web:reg]timeseries filters package Add-Inn written by Kurt Annen(2005). This is freeware programme.

0	-1.261	0.99971	2.374	1.730

KBL_hp: ADF tests (T=2617, Constant+ Trend; 5%=-3.41 1%=-3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-1.984	0.99954	2.365	1.722

KR_hp: ADF tests (T=2740, Constant;5%=-2.86 1%=-3.44)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-0.4374	0. 99992	4.852	3.160

KR_hp: ADF tests (9 T=2740, Constant+Trend; 5%=-3.41 1%=-3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	4.490	1.0009	4.766	3.124

PR_hp: ADF tests (T=2618, Constant; 5%=-2.86 1%=3.44)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-0.04069	0.99999	2.900	2.130

PR_hp: ADF tests (T=2618, Constant + Trend; 5%=3.41 1%=-3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	-2.715	0.99933	2.890	2.123

WIG_hp: ADF tests (T=3289, Constant; 5%=-2.86 1%=-3.44)

D-lag	t-adf	beta Y_1	sigma	AIC
0	11.75	1.0010	3.547	2.533

WIG hp: ADF tests (T=3289, Constant + Trend; 5%=-3.41 1%=-3.97)

D-lag	t-adf	beta Y_1	sigma	AIC
0	9.428	1.0014	3.541	2.530

As can be seen, the ADF test carried out for all the series with a removed stochastic trend allows us to reject the zero hypothesis about their nonstationarity. Therefore, the series obtained can be interpreted as stationary towards the stochastic trend. The graphs beneath include respective courses of the time series.

Removing the stochastic trend from the original data allows us to obtain the stationary series towards the trend, not towards the differences. Therefore, it is difficult to speak about the order of integration (or cointegration) of the researched processes. The attempts to select partial integrated processes (with the so-called fractional integration) are made more and more frequently [Geweke, Porter-Hudak, 1983; Robinson, 1995; Beyaert, 2003; Syczewska, 2004; Syczewska, 2005, pp. 122, 124, 128-135; Kufel, 2004, p. 70]. The order of integration *d* is, as we know, defined as the smallest integer of differentiations, after which the series can be treated as stationary[8]. So far it has been acknowledged that the order of integration can take only the integer values. Nowadays it is being acknowledged that the values *d* are the real numbers and they refer to the long memory processes (if $0.5 < d < 1$). The processes with non integer *d* are partially ("fractionally") integrated. If $d = 0$ then dis-

[8] "the nonstationary series which can be transformed into the stationary series, calculating the differences d times, is called the integrated series in order d", - [Charemza, Deadman 1997, s. 112].

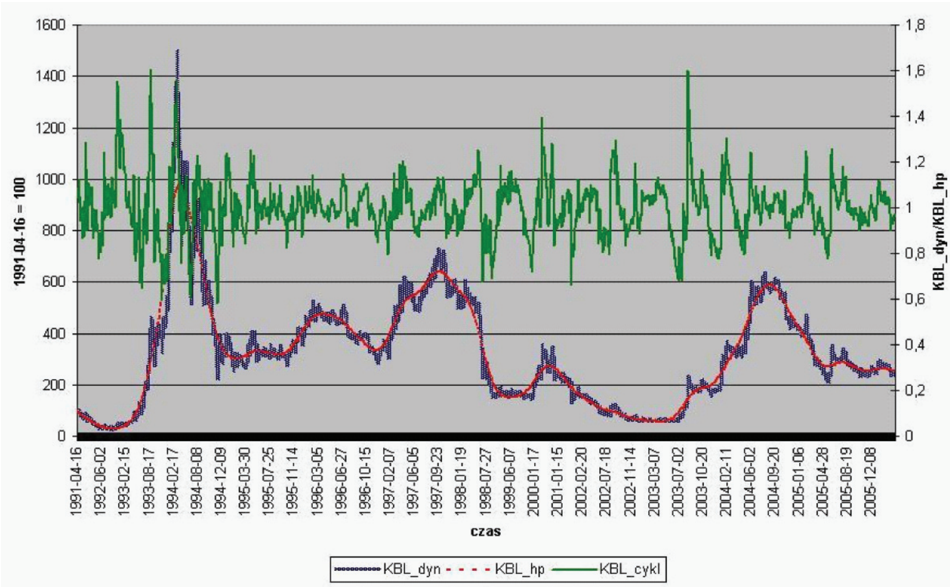

Figure 1.The time series of the quotations dynamics of Kable Company
Source: Self calculations

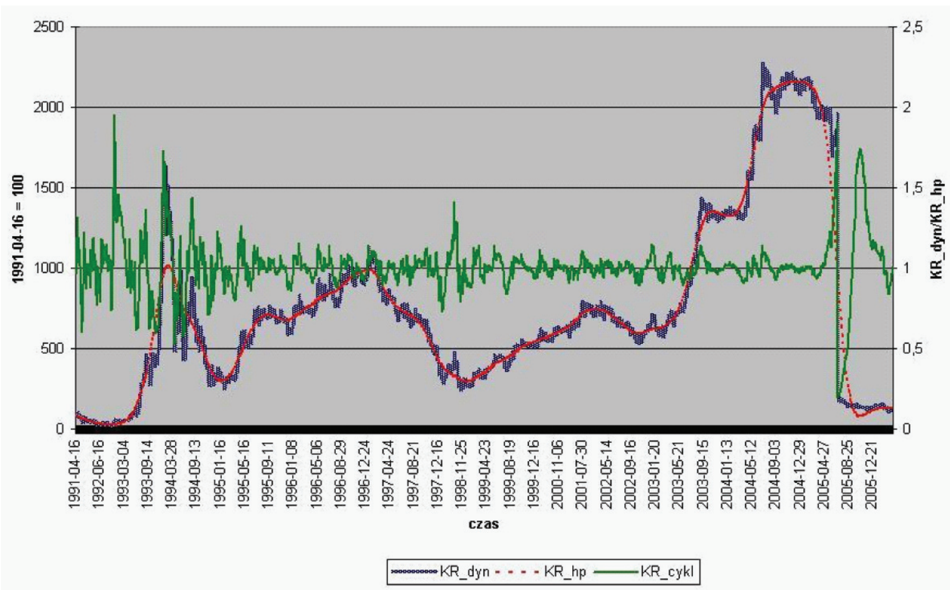

Figure 2. The time series of the quotations dynamics of Krosno Company
Source: Self calculations

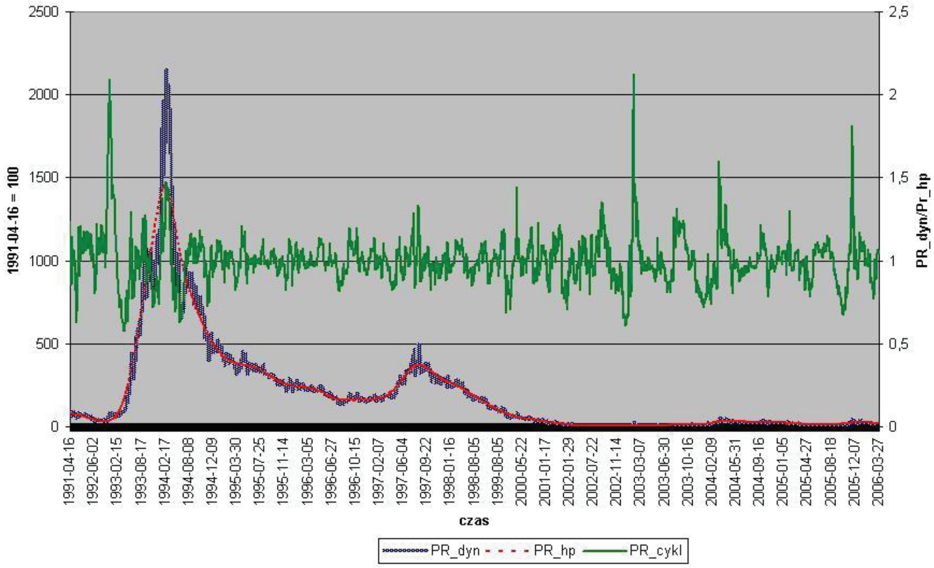

Figure 3. The time series of the quotations dynamics of Próchnik Company
Source: Self calculations

Figure 4. The time series of the quotations dynamics of WIG
Source: Self calculations

turbances (shocks) of the time series have the local character ("short memory") and their influence does not reveal itself in a long term. However, if $d>0$, the situation radically changes: disturbances can be transferred into the distant future and affect the shaping of the time series values. In this case one can speak about "long memory" or about the persistence of the time series [Peters, 1997, p. 244]. Permanence of the process memory depends on value d: the smaller d is, the less persistent the disturbances of the time series will be. If $d<0.5$ then the process can be acknowledged as the stationary one. Of particular interest for the economic time series is the case when $0.5<d<1$. This means that the series are simultaneously nonstationary and bestowed with the long "memory". Therefore, despite the occurrence of the phenomenon of disturbing data transmission into the distant future, the series reveals a tendency to keep the average dynamics at a fixed level for a long period of time. In other words, "the series has a long but *transitory memory*" [Beyaert, 2003, p. 3]. This is distinctly different from the situation when $d\geq1$, that means when the average does not display any long-term influence on the long-run series evolution. Such a series though, is dominated by the remote and the recent shocks (disturbances). This series is characteristic of a permanent, infinite memory. If $d<1$, we cannot exclude the existence of the long-run equilibrium (represented by the average). Simultaneously, it is obvious that for $0<d<1$ the traditional testing does not work, i.e. testing for the presence of the unit root (H_0: $d=1$) or the stationarity tests (H_0: $d=0$) developed basing on the works by Dickey-Fuller and Philips-Perron. In such conditions these tests are too weak to prove the existence of the series average and they lead to a false conclusion about the infinite memory of the researched process.

Therefore, let the process Y_t be generated by the model $(1-L)^d(Y_t-Y_0)=u_t$, where Y_0 is the random average of a given density of distribution, d does not have to be the integer, and u_t is the stationary process with the zero average. Such a process Y_t is defined as fractionally integrated in the order d. This process will be stationary and irreversible when and nonstationary when $d\geq0.5$. Until d<1 the process will have a tendency to keep the average at the same level. This means that the occurring disturbances will not permanently influence the future behaviour of the process. If $d=0$ the process dynamics is related to the stationary dynamics of the process u_t (such processes are characterized by the short memory). However if $d>0$ the process dynamics will depend, on the one hand, on the short-run dynamics u_t and on the other hand, on the long-run correlations occurring inside the series. Such a process is characterized by the long memory. The memory is transitory when $d<1$, or permanent when $d\geq1$. The value d is therefore a measure of the degree of persistence of disturbances: the lower this value is, the lower is the shock persistence. Establishing the value d is therefore of first-rate importance for the forecasting the economic processes. Particularly important is the answer to the question whether $d<0.5$, $0.5\leq d<1$ or $d\geq1$.

Various methods are used to estimate the partial (fractional) integration parameters. One of the most widely used is the Geweke and Porter–Hudak method, [Geweke, Porter-Hudak, 1983; Gil-Alana, 2000; Banerjee, Urga, 2004]. This method is based on the regression of the logarithmic values of the periodogram of the time series and on the logarithmic values of the Fourier frequency (GPH regression or LP regression). Another frequently used method is establishing the local Whittle's estimator [Künsch, 1987; Robinson, 1995 (1)]. Both methods demand removing the trend from the time series (nowadays, the methods of estimating d are being developed without differencing and without detrending the original data [Beyaert, 2003, p. 6]). P.M.Robinson proved the convergence and asymptotic normality of the LP and Whittle's estimators [Robinson, 1995; Robinson, 1995 (1); Deo, Hsieh, Hurvich, 2005]. Below there are the results of estimating the parameters of fractional integration using the Geweke and Porter-Hudak method and using the procedure of Whittle's local estimator. Both methods use the periodogram and the spectral density to estimate the real degree of integration.

Periodogram for the variable KBL_dyn
The number of observations = 2618
Test for the fractional integration (Geweke, Porter-Huduk)
The estimated fractional integration = 1,094 (0,132314)
the test statistics: t(50)=8,26824, with the value p 0,0000
The local Whittle's estimator (T=2618, m=111)
The estimated fractional integration=0,964837 (0,474579)
the test statistics: z = 20,3304, with the value p 0,0000

The periodogram for the variable KBL_hp
The number of observations = 2618
The test for the fractional integration (Geweke, Porter- Hudak)
The estimated fractional integration = 1,72093 (0,147608)
the test statistics: t(50) = 11,6587, with the value p 0,0000
The local Whittle's estimator (T= 2618, m = 111)
The estimated fractional integration = 1,73646 (0,0474579)
the test statistics: z = 36,5895, with the value p 0,0000

The periodogram for the variable KBL_cycle
The number of observations = 2618
The test for the fractional integration (Geweke, Porter-Hudak)
The estimated fractional integration = -0,288865 (0, 114287)
the test statistics: t(50) = -2,52754, with the value p 0,0147
The local Whittle's estimator (T = 2618, m = 111)
The estimated fractional integration = 0,392937 (0,0474579)
the test statistics: z = 8,2797, with the value p 0,0000

The periodogram for the variable KR_dyn
The number of observations = 2741
The test for the fractional integration (Geweke, Porter – Hudak)

The estimated fractional integration = 1,14947 (0,123683)
the test statistics: t(51) = 9,29375, with the value p 0,0000
The local Whittle's estimator (T = 2741, m = 114)
The estimated fractional integration = 1,01028 (0,0468293)
the test statistics: z = 21,5736, with the value p 0,0000
The periodogram for the variable KR_hp
The number of observations = 2741
The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = 2,31278 (0,146764)
the test statistics: t (51) = 15,7585, with the value p 0,0000
The local Whittle's estimator (T = 2741, m = 114)
The estimated fractional integration = 2,74053 (0,0468293)
the test statistics: z = 58,5216, with the value p 0,0000

The periodogram for the variable KR_cycle
The number of observations = 2741
The test for the fractional integration (Geweke, Porter –Hudak)
The estimated fractional integration = 0,0890018 (0,121581)
the test statistics: t(51) = 0,732039, with the value p 0,4675
The local Whittle's estimator (T = 2741,m = 114)
The estimated fractional integration = 0,535201 (0,0468293)
the test statistics: z = 24,6079, with the value p 0,0000

The periodogram for the variable PR_dyn
The number of observations = 2619
The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = 0,914973 (0,0505188)
the test statistics: t(50) = 18,1115, with the value p 0,0000
The local Whittle's estimator (T = 2619, m = 111)
The estimated fractional integration = 1,16784 (0,0474579)
the test statistics: z = 24,6079, with the value p 0,0000

The periodogram for the variable PR_hp
The number of observations = 2619
The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = 2,10974 (0,112251)
the test statistics: t(50) = 18,7949, with the value p 0,0000
The local Whittle's estimator (T = 2619, m = 111)
The estimated fractional integration = 2,23922 (0,0474579)
the test statistics: z = 47,1833, with the value p 0,0000

The periodogram for the variable PR_cycle
The number of observations = 2619

The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = -0,0995222 (0,106873)
the test statistics: t(50) = -0,931216, with the value p 0,3562
The local Whittle's estimator (T = 2619, m = 111)
The estimated fractional integration = 0,467854 (0,0474579)
the test statistics: z = 9,8583, with the value p 0,0000

The periodogram for the variable WIG_dyn
The number of observations = 3290
The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = 0,940674 (0,0701059)
the test statistics: t(50) = 13,4179, with the value p 0,0000
The local Whittle's estimator (T = 3290, m = 127)
The estimated fractional integration = 1,00747 (0,0443678)
the test statistics: z = 22,7072 with the value p 0,0000

The periodogram for the variable WIG_hp
The number of observations = 3290
The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = 0,909616 (0,0359514)
the test statistics: t(56) = 25,3013, with the value p 0,0000
The local Whittle's estimator (T = 3290, m = 127)
The estimated fractional integration = 1,01006 (0,0443678)
the test statistics: z = 22,7656 with the value p 0,0000

The periodogram for the variable WIG_cycle
The number of observations = 3290
The test for the fractional integration (Geweke, Porter – Hudak)
The estimated fractional integration = -0,483769 (0,0872957)
the test statistics: t(56) = -5,54173, with the value p 0,0000
The local Whittle's estimator (T = 3290, m = 127)
The estimated fractional integration = 0,172351 (0,0443678)
the test statistics: z = 3,88461, with the value p 0,0001

The results will be presented in the table and on the graph, where d_{GPH} is the estimator of the Geweke Peters- Hudak fractional integration, d_W – the Whittle's local estimator, σ^2– the standard error, p – the empirical level of significance. The analysis of the results obtained allows to establish the short memory of the stationary time series deprived of the stochastic trend, the permanent memory of the non stationary data of the Hodrick-Prescott trend and the declining memory of the non stationary time series of the quotation dynamics of the stock companies. It is characteristic that for the non stationary series of the WIG aggregation, the estimators of the fractional integration values, are close to one, which would testify to its smaller persistence as regards the companies' quotations.

Table 1. The fractional integration estimators by Geweke, Peter-Hudak and Whittle

Market quotations	Estimators d	σ^2	p
KBL_dyn d_{GPH} d_W	 1.094 0.964837	 0.132314 0.0474579	 0.0000 0.0000
KBL_hp d_{GPH} d_W	 1.72093 1.73646	 0.147608 0.0474579	 0.0000 0.0000
KBL_cycle d_{GPH} d_W	 -0.288865 0.392937	 0.114287 0.0474579	 0.0147 0.0000
KR_dyn d_{GPH} d_W	 1.14947 1.01028	 0.123683 0.0468293	 0.0000 0.0000
KR_hp d_{GPH} d_W	 2.31278 2.74053	 0.146764 0.0468293	 0.0000 0.0000
KR_cycle d_{GPH} d_W	 0.0890018 0.535201	 0.121581 0.0468293	 0.4675 0.0000
PR_dyn d_{GPH} d_W	 0.914973 1.16784	 0.0505188 0.0474579	 0.0000 0.0000
PR_hp d_{GPH} d_W	 2.10974 2.23922	 0.112251 0.0474579	 0.0000 0.0000
PR_cycle d_{GPH} d_W	 -0.0995222 0.467854	 0.106873 0.0474579	 0.3562 0.0000
WIG_dyn d_{GPH} d_W	 0.940674 1.00747	 0.0701059 0.0443678	 0.0000 0.0000
WIG_hp d_{GPH} d_W	 0.909616 1.01006	 0.0359514 0.0443678	 0.0000 0.0000
WIG_cycle d_{GPH} d_W	 -0.483769 0.172351	 0.0872957 0.0443678	 0.0000 0.0001

Source: Self calculations based on GNU Regression Econometrics Time-Series Library-gretl ver.1.5.1 (GNU GENERAL PUBLIC LICENSE, http://gretl.sourceforge.net).

The statement made by Kolmogorow-Cramer says: if the real process can be presented as the stationary stochastic process in a broad sense (of zero average value) then this process can be presented in a spectral form [Swiesznikow, 1965, chapter II; Gichman, Skorochod, 1968, chapter I; Talaga, Zieliński, 1986, pp. 18-25; Stoica,

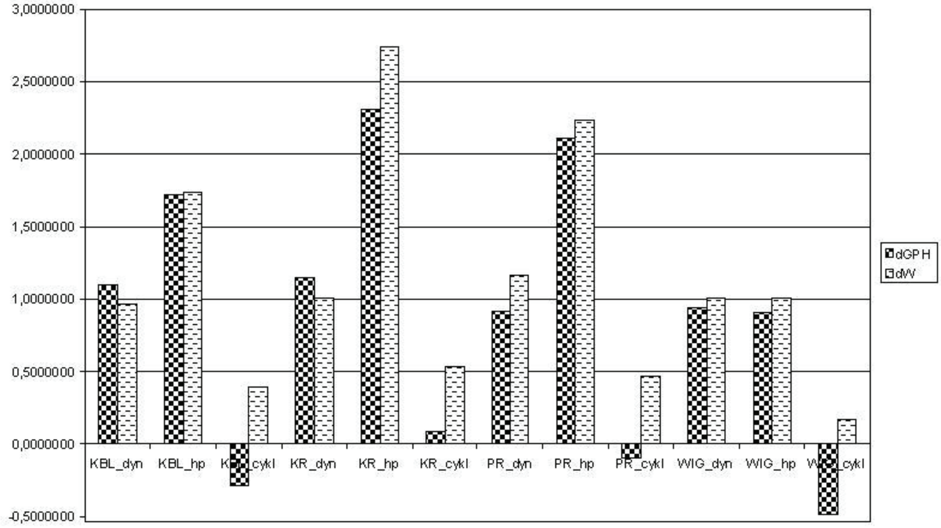

Figure 5. Estimators of long memory by Geweke, Peter-Hudak and Whittle
Source: Self calculations (see Table1)

Moses, 1997, pp. 6-13]. The figures below illustrate the functions of spectral density of the stationary (towards the stochastic trend) and the nonstationary time series. The general course of the spectral density curves is typical for the stationary, stochastic economic series. The curves take the highest values for low frequencies and gradually decrease simultaneously with a rise of the frequencies of harmonious fluctuations occurring in the time series. The most important share in the variability of the researched processes have the long run fluctuations. With respect to the stock quotations these are the fluctuations with the periods about 34, 20 and 14 stock sessions. The short run fluctuations for the data of the Hodrick-Prescot stochastic trend indicate a relatively large share in the general variance of the series. The specification of the spectral density function presented on the graphs below shows a high convergence in the harmonic structures of the stationary and nonstationary processes in the low frequency area. This convergence is considerably weaker for the high frequencies. This would be an important guideline in the construction of a dynamic econometric model of the stock quotations: it is easier to obtain the congruent, linear, dynamic models for the long run fluctuations than for the short run fluctuations. In his work devoted to the selected characteristics of the time series of economic data C.W.J.Granger in 1981 proved the inconsistency of the model with the seasonal explained variable, non-seasonal explaining variables and the random residual process [Granger, 1981]. On the graphs below, marked with the arrows are the short run fluctuations, giving their periodicity (calculated as the reverse of frequency) in the number of the stock sessions together with the companies quotations and WIG quotations. The values of the periods described in the arrows text fields

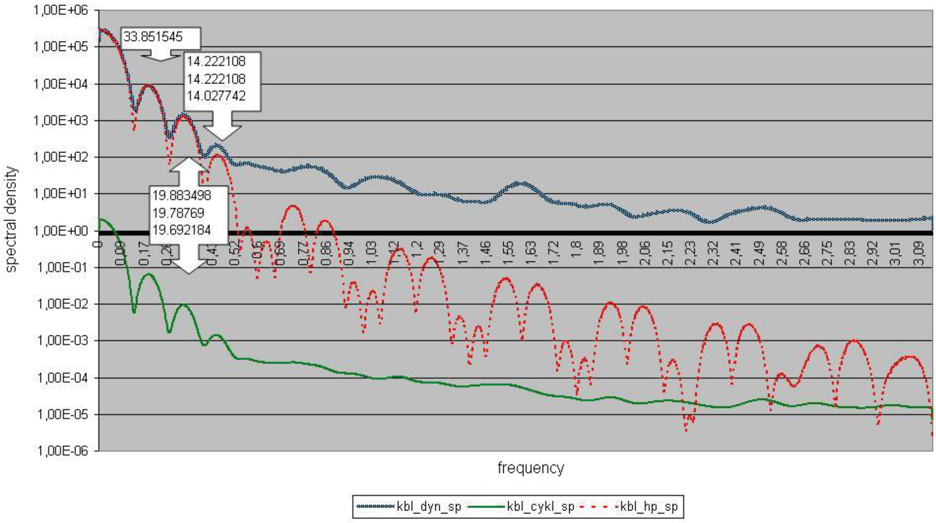

Figure 6. The Tukey spectrum dynamics time series of the stock index of Kable company

Source: Self calculations in the Matlab environment (Signal Processing Toolbox), Copyright 20004, ver.7.0.1.24704 (R14) Service Pack1, Licence number 265559, the Math Works, Inc. MATLAB is a registered trademark of The Math Works, Inc.

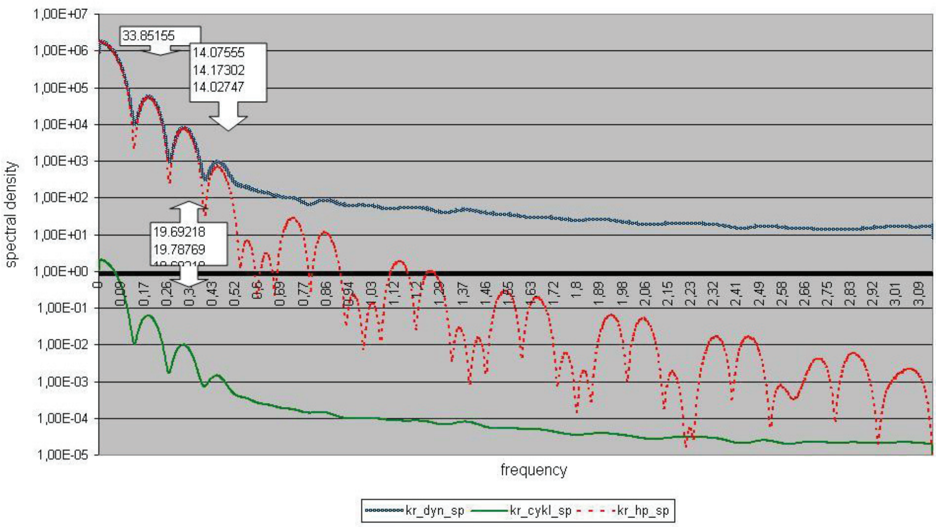

Figure 7. The Tukey spectrum dynamics time series of the stock index of Krosno company

Source: Self calculations (see Figure 6)

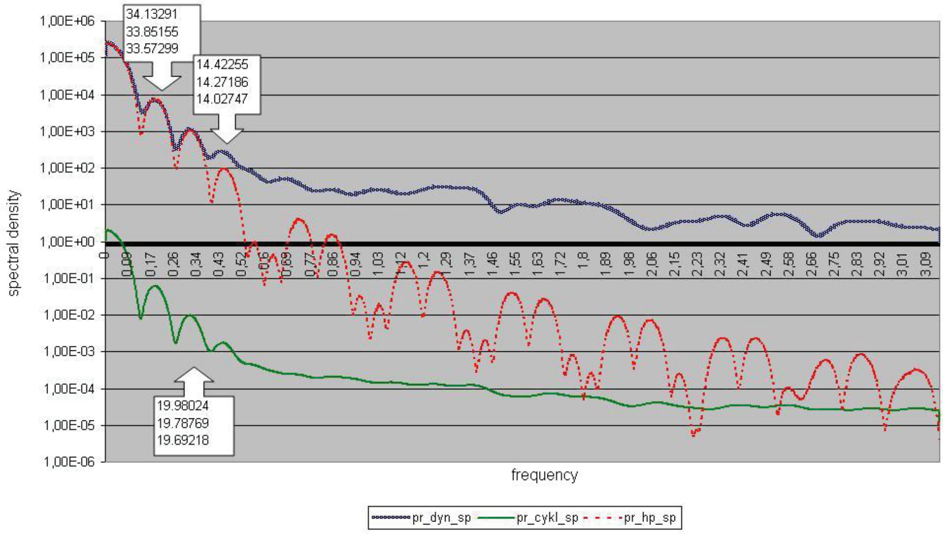

Figure 8. The Tukey spectrum dynamics time series of the stock index of Próchnik company

Source: Self calculations (see Figure 6)

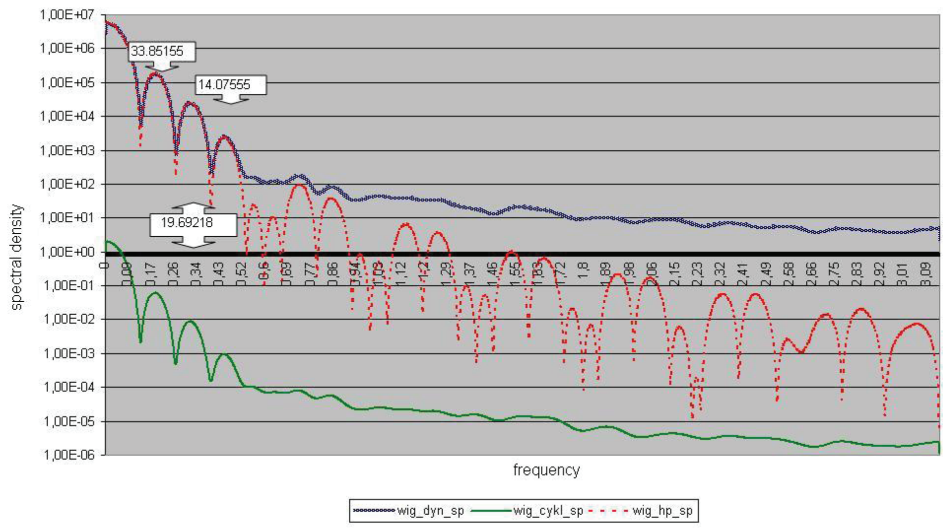

Figure 9. The Tukey spectrum dynamics time series of the Warsaw Stock Index (WIG)

Source: Self calculations (see Figure 6) concern – respectively – the "raw" indicators of dynamics (_dyn), the stochastic trend indicators (_hp) and the cyclical element indicators (_cycle).

As results from the research, the spectral analysis of the stock quotations does not reveal the fluctuations in the market conditions. The fluctuations with the period of about 34, 20 and 14 sessions with the quotations of particular companies and WIG correspond to the periods 8-10, 5-6 and 3-4 weeks[9]. The lack of business fluctuations is conditioned, on the one hand, by the rigorous assumptions of the spectral analysis as regards the properties and the length of the time series and, on the other hand, by a specific character of the Stock Market in Poland. This market is characterized, among others, by a high level of concentration, a low level of turnover and, above all, institutional underdevelopment. A characteristic feature of the rates quoted on the Warsaw Stock Exchange is high demand for the stocks (mainly ordered by the Investment Funds Associations and by the Open Pension Funds) and not a very big supply (a small number of Stock debuts). Forecasting the dynamics of the stock exchange rates is, in these conditions, very difficult. The Warsaw Stock Exchange, as regards the value of the Stock debuts in the fourth quarter of 2006[10] ranked 7th in Europe (this is hardly 3% compared to 40% of the London Stock Exchange, 30% of Euronext and 12% of Deutsche Börse). The share of institutional investors in the volume of the stock turnover showed a strong dynamics in the period 1997- 2006 and increased from 24% in 1997 to 39 % in 2002 in 2003:

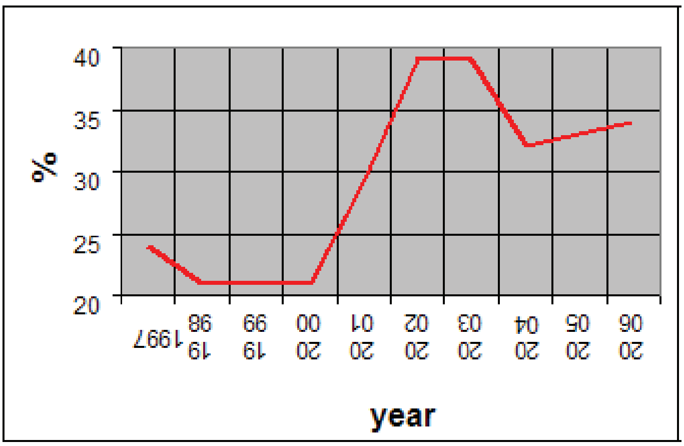

Figure 10. The share of the institutional investors in the stock turnover volume, in general (in %)

Source: Self calculations based on www//gpw.com.pl/źródła/gpw/prezentacje

[9] In connection with a different frequency of sampling the Stock Exchange quotations, the fluctuation periodicity measured by the number of sessions was corrected so as to obtain the periodicity measured by time. The 7 sessions period is the longest for the series with the lowest frequency of probation of Kable and Próchnik, a little bit shorter for Krosno and the shortest for WIG.

[10] In accordance with the PricewaterhouseCoopers inquiry, www.pwc.com/pl/pl/about/.

To obtain the answer to the question about the existence of the longer- run fluctuations, we will refer to the analysis of the rescaled range. Therefore, we will try to define the maximum range of "memory" for the particular economic processes.The rescaled range analysis, R/S used for the research on the time series of the stock indexes allows us to define the random walking bias, i.e. to measure the trend's strength and the level of the disturbing "noise" in them. The measure is deviation of the Hurst coefficient from 0.5 [Peters, 1997, p.65]. The Hurst coefficient takes the following values: (1) H=0.5, (2) 0≤H< 0.5 and 0.5<H≤1. In the first case the events are of the random character and they are not correlated. In the second case we deal with the antipersistent systems. The time series in this case are more variable than the random series. The processes they describe are characterized by the behaviour defined in the literature as "returning to a mean". After each strong deviation is followed by opposite (i.e. equally strong), balancing deviation. The closer H is to zero, the more ergodic the process is. However, in the third case, when 0.5<H≤1 we deal with the long memory processes, persistent ones, supporting the trend. If H is closer to one, the stronger is the power of behaviours reinforcing the trend. The closer H is to 0.5, the higher is the level of the "noise" and the more fuzzy is the trend. The influence of the current phenomena on the future phenomena can be defined by the correlation coefficient: $C=2^{(2H-1)}-1$. For H=0 the correlation C=-½, for H=1 C=1, which means that the process has an infinite memory and it resembles the situation described by Koheleth:

"That which hath been is that which shall be,

- and that which hath been done is that which shall be done;
- and there is nothing new under the sun" [Bible, Ecclesiastes 1.9; Tako głosi Kohelet, 2000, p. 28].

The results obtained prove the persistence of the series of stock quotations dynamics. H is bigger than 0.5 and the probability density function of these series is not the curve of the normal density. If H=0.5 it does not mean that we deal with the Gauss random walk; this only means that the analyzed process is deprived of the long memory. As regards the persistent time series fractal densities of probability apply. The measure of the fractal density is the so-called fractal dimension (D). It is connected with the Hurst coefficient: D=2-H. These processes are a mixture of behaviours consistent with the trend and the recurrent, cyclical behaviours. The cyclical ones can be roughly estimated. Roughly, because we deal with the fractal and non periodic processes. The length of the cycle defines the range after which the process loses the memory and behaves ergodically. In the present research the average period of the cycle was determined in the number of daily quotations. This number is not the same for all the companies and for WIG, although it covers the same period of almost 15 years, from 16th April 1991 to 27th March 2006. Over this period the quotations number of Kable was 2618, Krosno – 2741, Próchnik – 2619 and WIG- 3290. The results of the analysis of the rescaled range allow us to formulate the thesis about occurrence of the long and the short term cycles. The short cycles, after which the systems behave ergodically (H=0) equal: for Kable 40 (repeated 63 times), for Krosno 60 (repeated 42 times) and for WIG 64 (repeated 45 times) of the daily quotations, i.e. from 12 to 18 weeks. However, the long cycles, after which the system loses its memory completely and behaves erratically and unpredictably, are respectively: 252 (10 cycles), 315 (8 cycles), 288 (10 cycles) and for Próchnik 504 (5 cycles) of the quotations, which would correspond approximately to 18, 21, 16 and 36 months. Forecasting the stock quotations indexes after these periods can be defined as Koheleth beholds: "vanity and a striving after wind" [Bible, Ecclesiastes 1.14; Tako głosi Kohelet, 2000, p. 29]. The Hurst coefficients, the fractal dimensions, the correlation coefficients and the approximate cycles (on the quotation days) were shaped as follows:

Table 2. The R/S analysis of selected quotations

Daily quotations	Hurst coefficient	Fractal dimension	Correlation coefficient	Cycles on quotations days
Short cycles				
Kable	0.553	1.447	7.6%	40 (12 weeks)
Krosno	0.545	1.455	6.4%	60 (18 weeks)
WIG	0.561	1.439	8.8%	64 (16 weeks)
Cykle długie				
Kable	0.599	1.401	14.7%	252 (18 months)
Krosno	0.564	1.436	9.3%	315 (21 months)
Próchnik	0.571	1.429	10.3%	504 (36 months)
WIG	0.607	1.393	16%	288 (16 months)

Source: Self calculations in BENOIT programme Ver. 1.31, TruSoft Inf'l, Inc. Copyright 1997, 999.

The R/S analysis was then illustrated by means of the double logarithmic graphs below. The index by the x variable in the regression equations is the Hurst coefficient on the graphs.

For the fractal processes *the self similarity* is the characteristic property. We will try to answer to the question whether the researched processes have such a feature. Intuitively, the idea of self similarity does not create any difficulties. It is an extension of the notion known from geometry: similarity: "Two objects, regardless of their sizes, are similar, if they are of the same shape … Similarities are the transformations permitting the homothety, rotations and shifts" [Peitgen, Urgens, Saute, 1995, p. 188]. Fractals as self similar objects consist of subsequent generations of similarly shaped objects. In the non mathematical fractals the diminished copies are not identical with the whole, but they reveal some deviations. In this case we deal with the stochastic or statistic self similarity [Peitgen, Urgens, Saute, 1995,

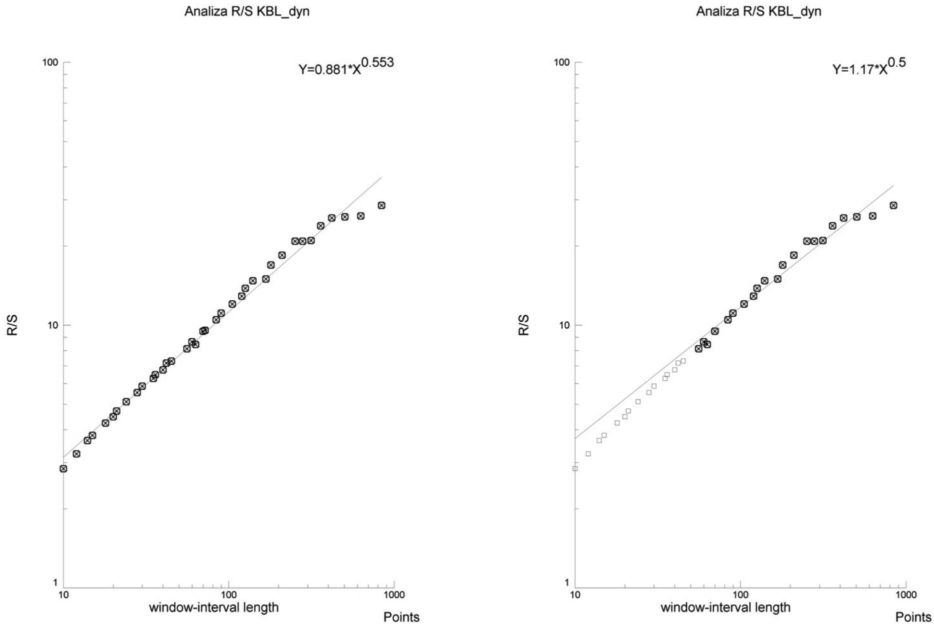

Figure 11. The R/S analysis of Kable (short cycles)
Source: Self calculations, see Table 2

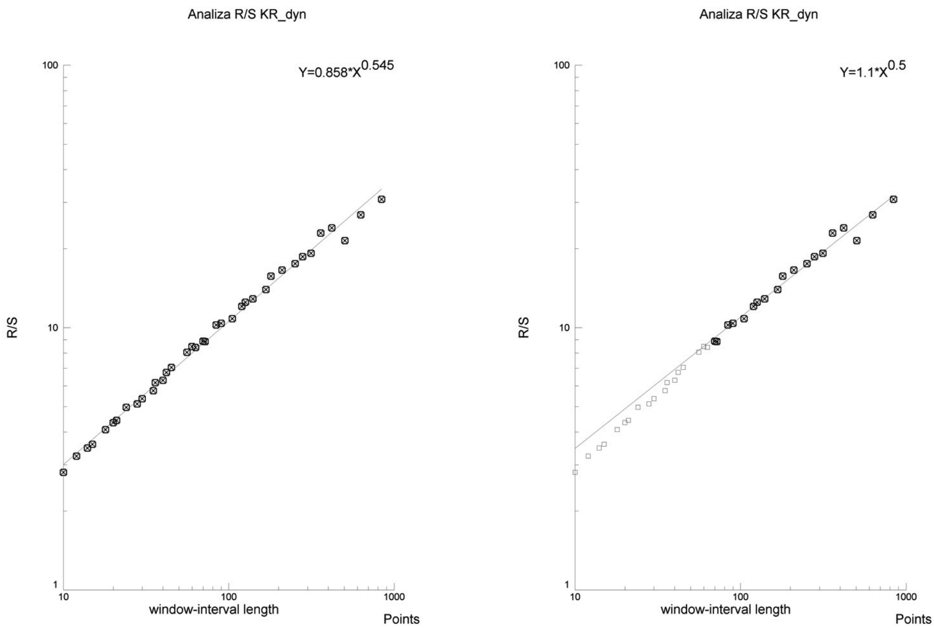

Figure 12. The R/S analysis of Krosno (short cycles)
Source: Self calculations, see Table 2

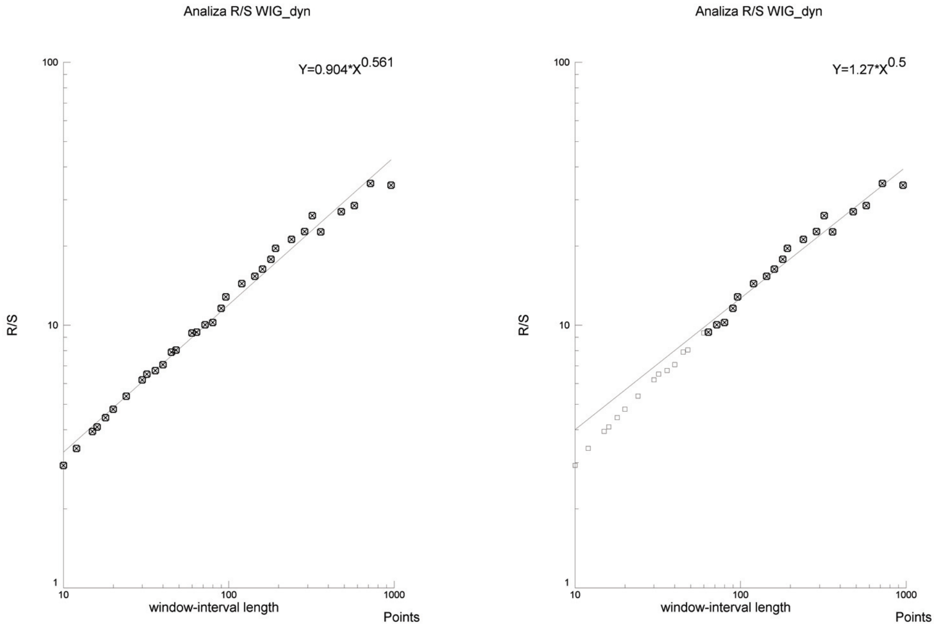

Figure 13. The R/S analysis of WIG (short cycles)
Source: Self calculations, see Table 2

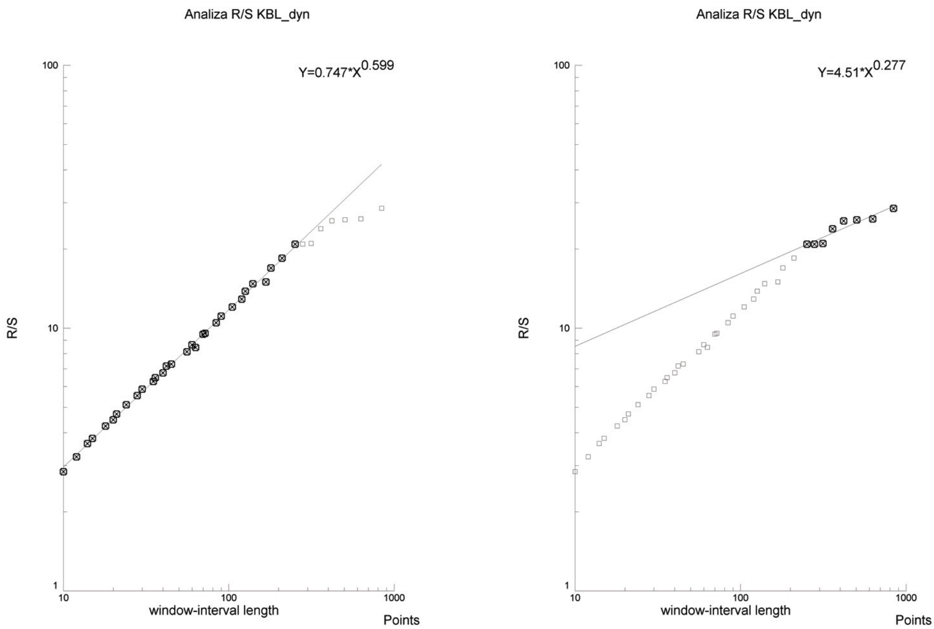

Figure 14. The R/S analysis of Kable (long cycles)
Source: Self calculations, see Table 2

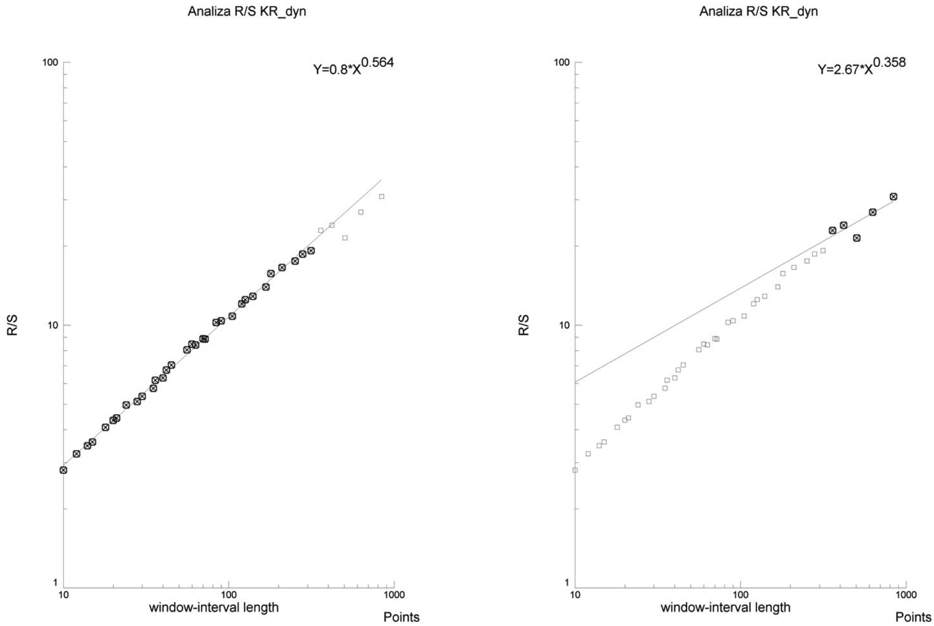

Figure 15. The R/S analysis of Krosno (long cycles)
Source: Self calculations, see Table 2

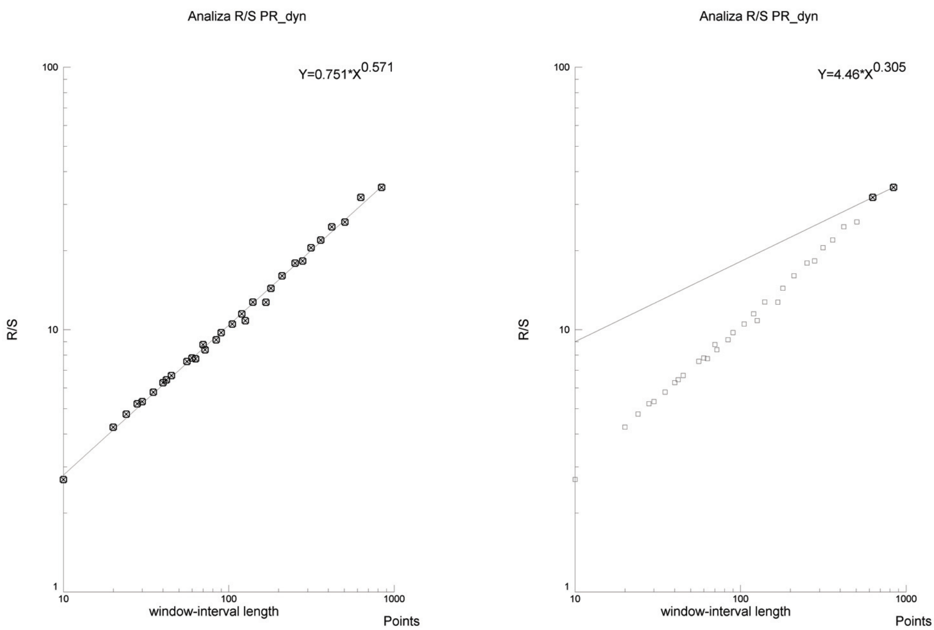

Figure 16. The R/S analysis of WIG (long cycles)
Source: Self calculations, see Table 2

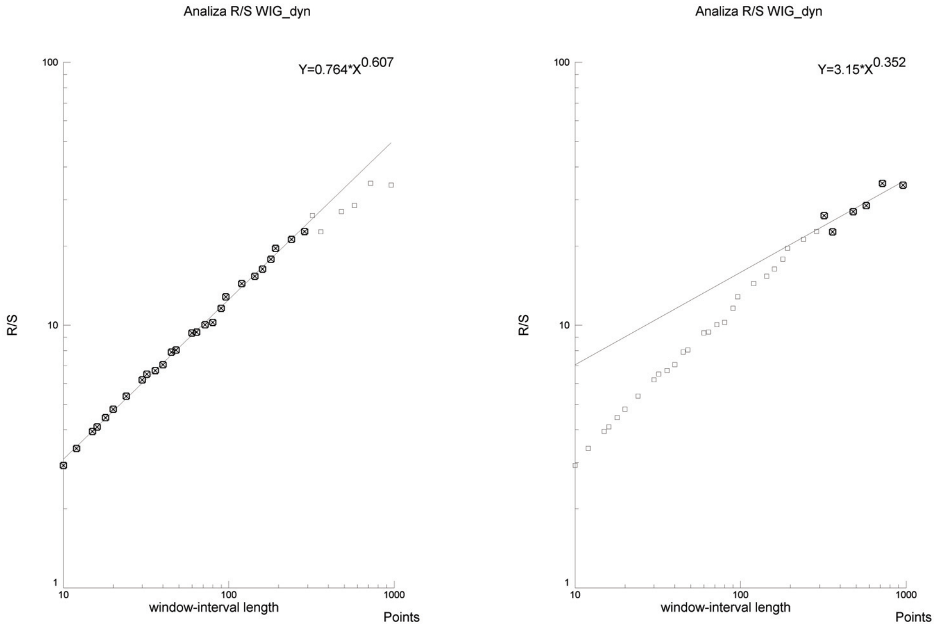

Figure 17. The R/S analysis of WIG (long cycles)
Source: Self calculations, see Table 2

p. 197].To establish the self similarity of the stationary time series (signals) we use the one-dimension, continuous wavelets analysis [Białasiewicz, 2000, pp. 219-221]. A continuous transformation of the wavelet function $f(t) \in L^2$ is described by the following equation:

$$Wf(b,a) = (f, \psi_{ab}) = \int_{-\infty}^{\infty} f(t)\psi_{ab}(t)^* \, dt, \tag{4}$$

where *a* denotes the scale, *b* – shift, *Wf* (b,a) – the wavelet coefficients (they are the function of position and scale).

A change in the scale means expansion (increasing the scale) or compression (decreasing the scale) of the wavelet. The small scale means that *Wf* are represented by the low frequency elements, however, the big scale means that *Wf* describe the high frequency elements of the signal. As the maximum level of the scale we regard the moment of losing the long term memory (it is linked with the lowest frequency obtained in the R/S analysis). On the graphs below the wavelet analysis reveals the feature of self similarity of the stationary time series not in the whole researched period, but in its sub-periods (marked with arrows). The time series of Kable stock index reveals the self similar structure: from May 1994 to June 2005. Respectively, the self similarity of the stationary course of Krosno market quotations concerns the period from February 1994 to December 2005, Próchnik from January 1995 to May 2004 and WIG from April 1994 to December 2004. Therefore this is the period of about 10-11 years. The self similarity of other frequencies (other scales) of the cyclical fluctuations did not differ from the given picture. Therefore, we can presume that over the years 1994/95 and 2004/2005 fundamental changes took place in the capital market dynamics in Poland (see graphs 17-24).

The wavelet analysis offers the tools allowing to discover the moment of discontinuous, rapid change in the frequency of the signal. This is important for the forecasting of the capital market situation which is changing dynamically. Increase or decrease in the frequency of the signal correspond to compression or stretching of the period of the dynamics waving. Then the entities making decisions have the impression that time accelerates or slows down. A discontinuous change of the time series frequency is impossible to be detected by means of other methods [Białasiewicz, 2000, p. 218]. The analysis results are illustrated on the graphs 25-28. The moment of rapid change of the signal frequency is identified by the "needle" of the detail (the arrow with the date). The wavelets associated with the details (there can be more of them, but to simplify the analysis we accepted one detail) have a narrow spectrum of frequency of a high middle frequency. Such a wavelet is not associated with the

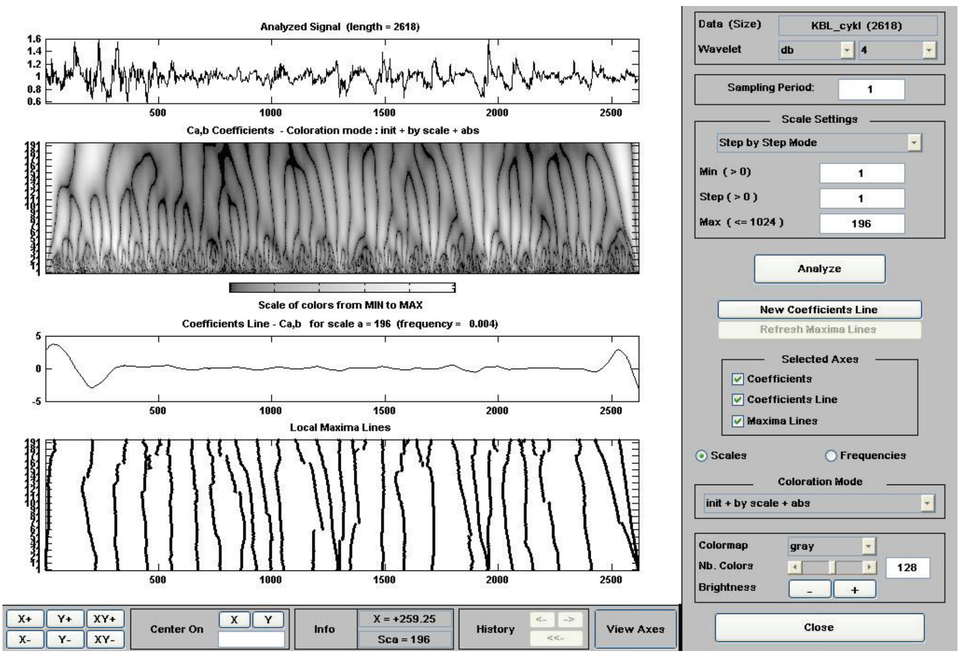

Figure 18. The self similarity of Kable quotations dynamics

Source: Self calculations in MATLAB environment (Wavelet Toolbox), Copyright 2004, ver. 7.0.1.24704 (R14) Service Pack 1, Licence number 265559, The Math Works, Inc. MATLAB is a registered trademark of The MathWorks, Inc.

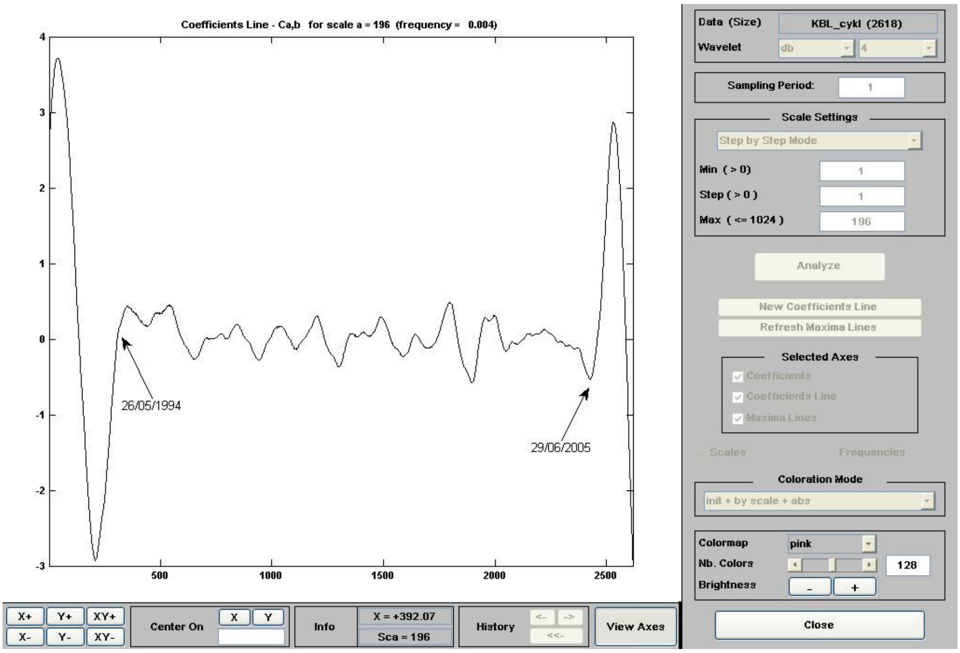

Figure 19. The line of wavelet coefficients of Kable quotations dynamics (a=196)

Source: Self calculations, see Figure 18

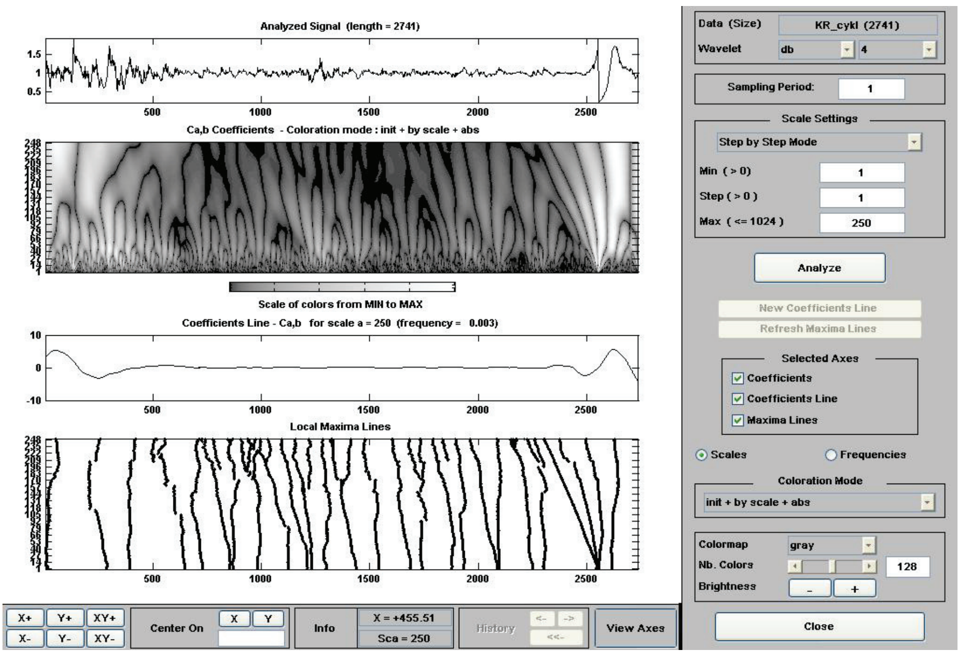

Figure 20. The self similarity of Krosno quotations dynamics

Source: Self calculations, see Figure 18

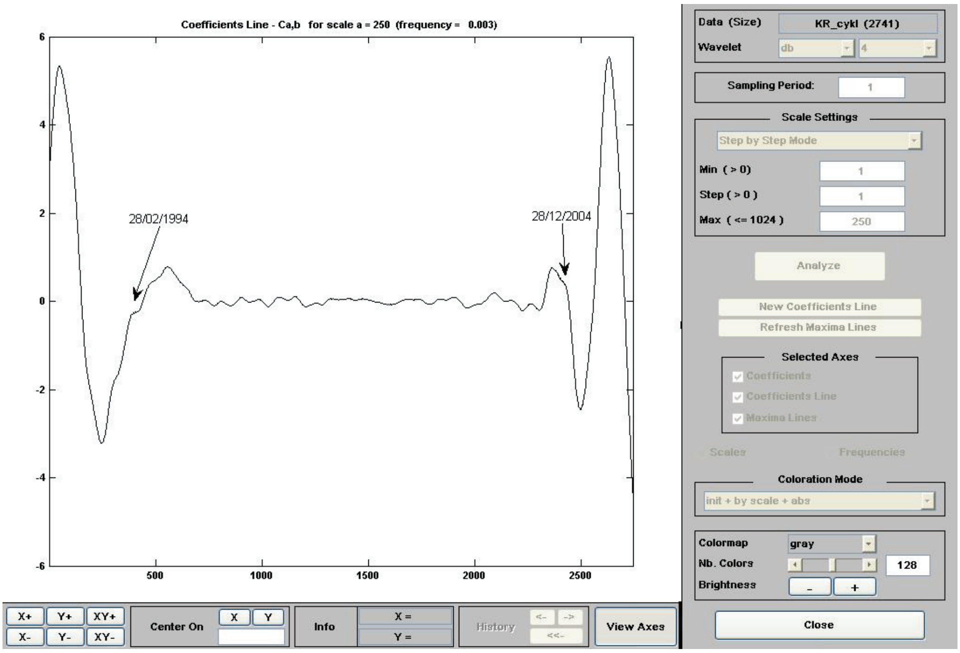

Figure 21. The line of wavelet coefficients of Krosno quotations dynamics (a=250)

Source: Self calculations, see Figure 18

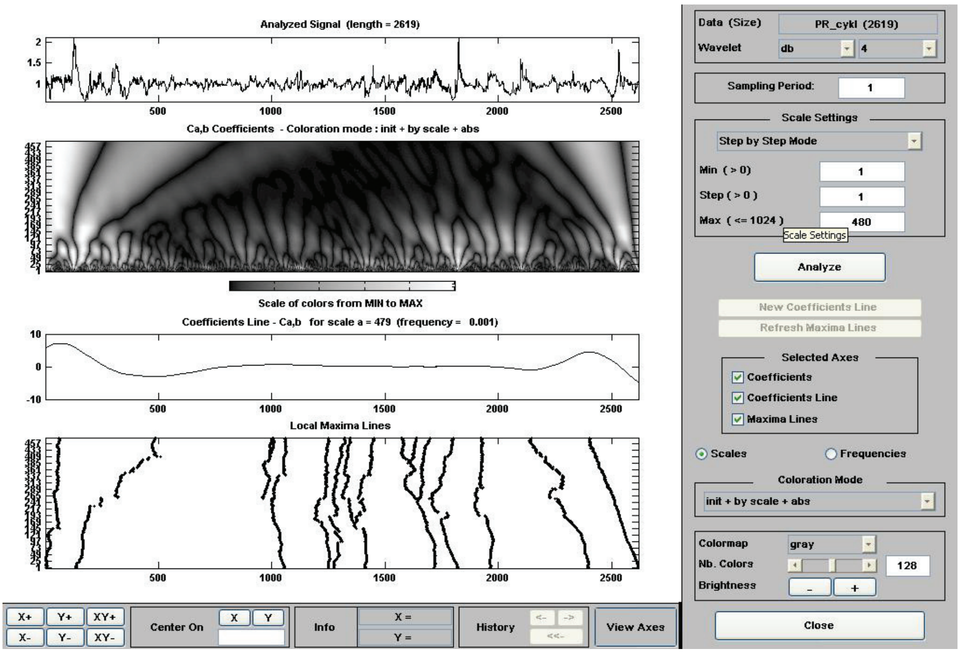

Figure 22. The self similarity of Próchnik quotations dynamics

Source: Self calculations, see Figure 18

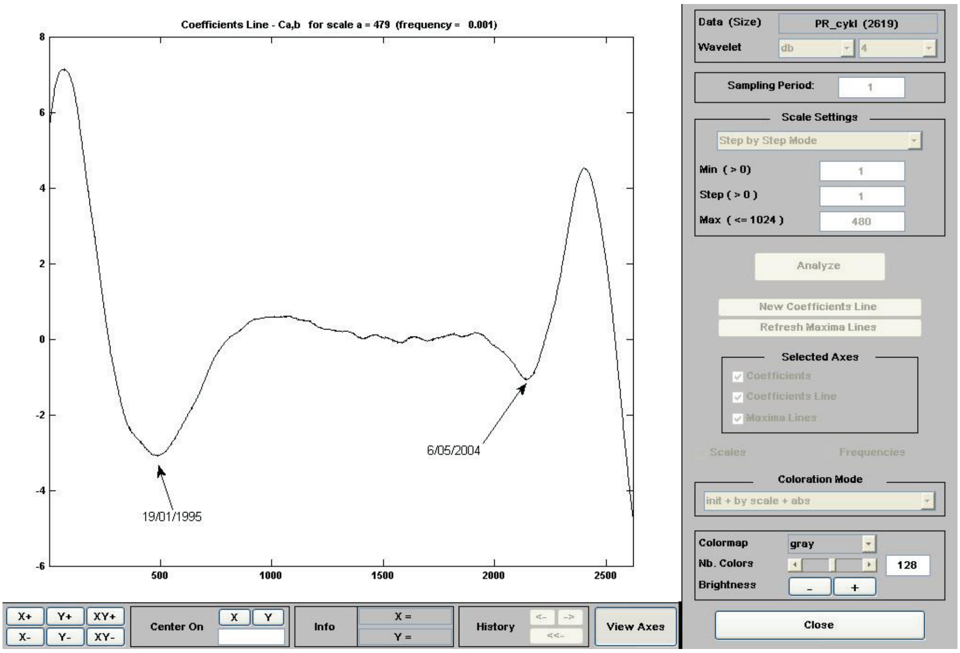

Figure 23. The line of wavelet coefficients of Próchnik quotations dynamics (a=480)
Source: Self calculations, see Figure 18

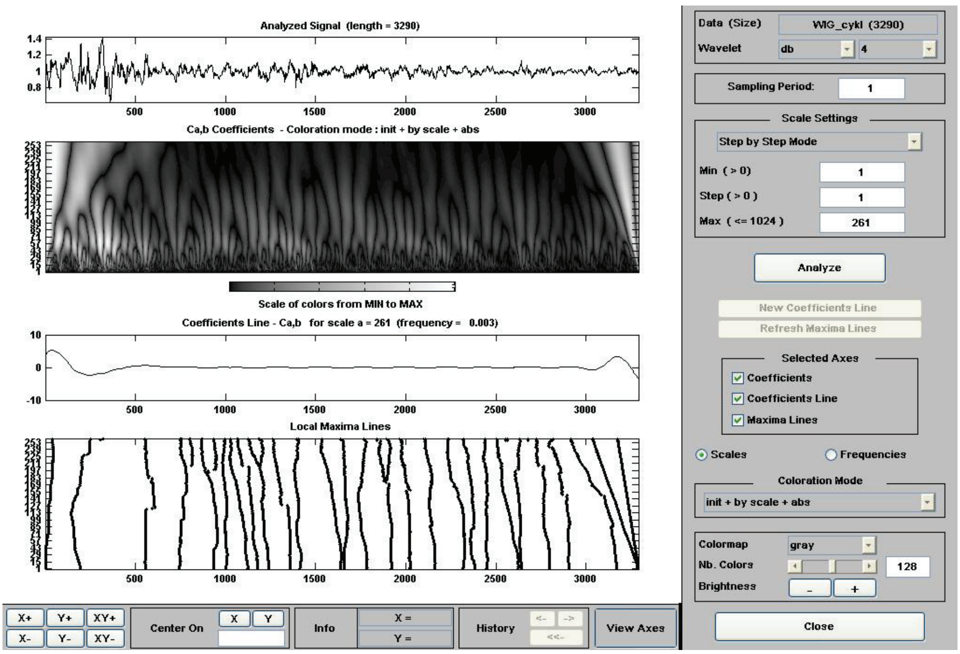

Figure 24. The self similarity of WIG quotations dynamics
Source: Self calculations, see Figure 18

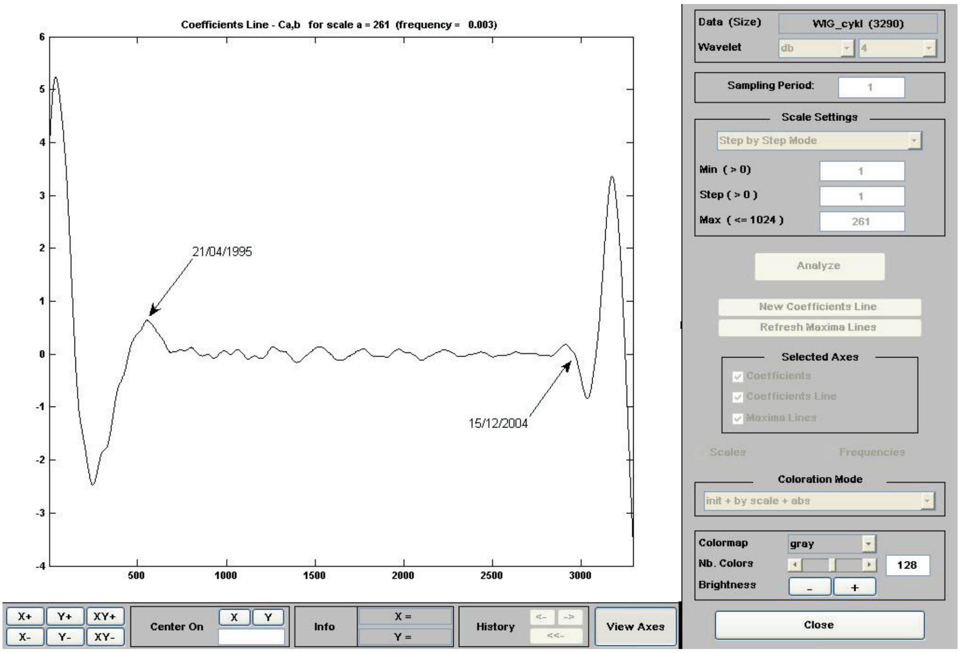

Figure 25. The line of wavelet coefficients of WIG quotations dynamics (a=261)
Source: Self calculations, see Figure 18

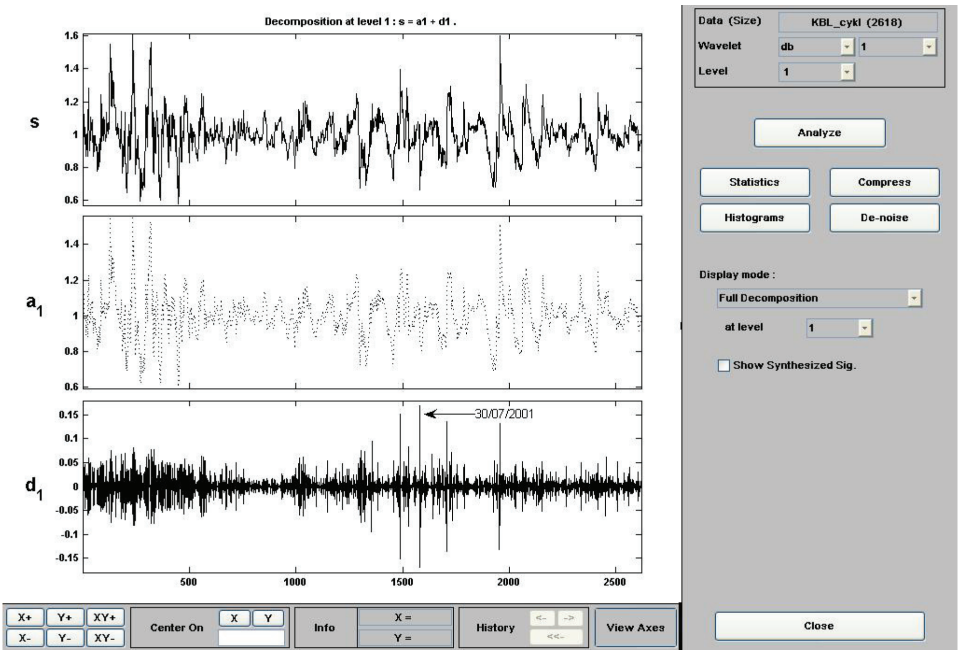

Figure 26. The discontinuous change of the frequency of Kable quotations dynamics
Source: Self calculations, see Figure 18

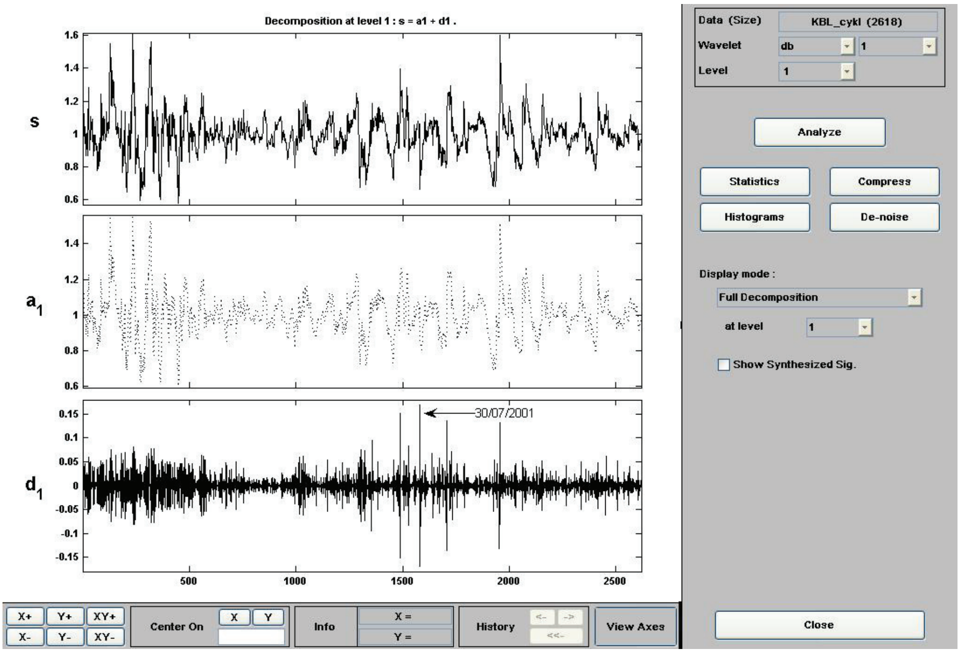

Figure 27. The discontinuous change of the frequency of Krosno quotations dynamics

Source: Self calculations, see Figure 18

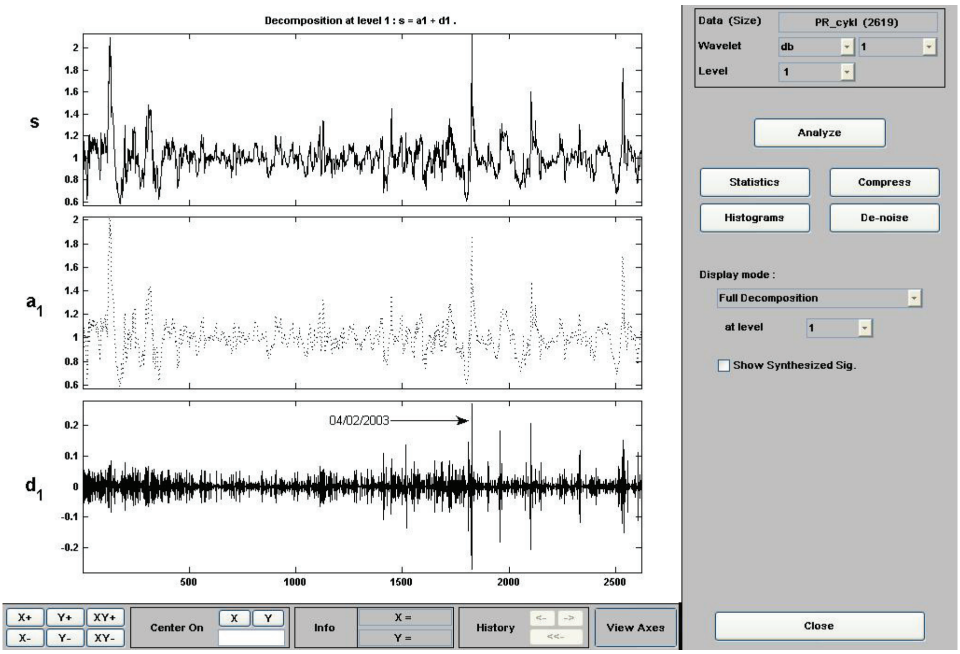

Figure 28. The discontinuous change of the frequency of Próchnik quotation dynamics

Source: Self calculations, see Figure 18

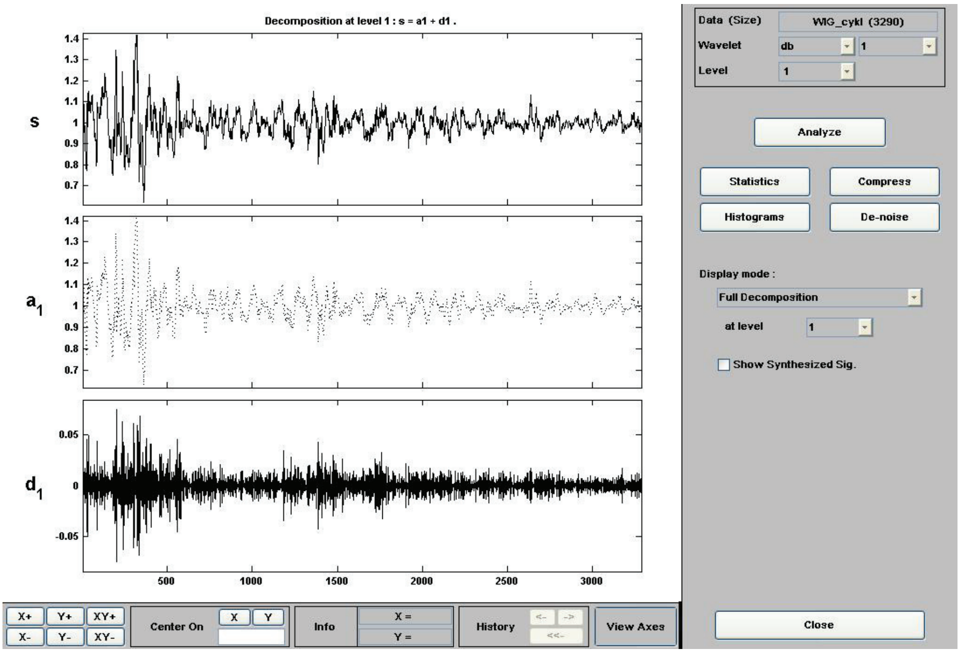

Figure 29. The discontinuos change of the frequency of WIG quotations dynamics
Source: Self calculations, see Figure 18

signal apart from the moment when a leap change of the signal frequency occurs. However, it is strongly correlated with the high frequency element, whereas. The Fourier transform does not demonstrate this advantage. Using it we cannot detect a separate event in the analyzed signal. The wavelet analysis is therefore an important supplement of the spectral analysis.

The time series of Próchnik trade quotations most clearly illustrate the change in the frequency of the stock quotations dynamics, whereas the changes in WIG quotation dynamics are the least visible. This results from the statistic, aggregated character of the Warsaw Stock Index. It does not represent any real economic process. As different processes clash there and the picture obtained is very fuzzy. Thus, the usefulness of WIG quotations for the forecasting of real economic processes seems to be doubtful.

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