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Alternative bargaining solutions in asymmetric tariff rates negotiations

Abstract: The article was dedicated to the application of cooperative games tools to the particular bargaining problem. The bargaining is about tariff rates between two countries. Analysis was performed on the framework of simple market model with perfect competition within countries and bilateral monopoly relation between them.
There were two bargaining schemes applied in order to calculate cooperative solutions. First was Nash bargaining solution, the second was Kalai and Smorodinsky proposition. Both methods successfully indicated cooperative solutions. Application of chosen bargaining schemes brought the conclusion that outcome of the indications of cooperative solutions strongly depends on the nature of explored economic model. The examination of influence parameters' changes proved that worsening the situation of the subject led to the decrease of its benefit in every case. In one case it also caused the decrease of benefit of the other party.
Keywords: cooperative games, market structure and pricing, Nash bargaining solution, Kalai and Smorodinsky bargaining solution.
Jel codes: C71, D40.

1. Introduction

The problem of bargaining always appears when the interests of economic parties are neither strictly opposite nor strictly consistent. Unanimous choice of cooperative solution gives more benefits to the players than mutual choice of non-cooperative solution. The possibility of direct negotiations between parties creates cooperative nature of the game. The aim of negotiations is to reach a binding agreement which points out a cooperative solution. The agreement is "guarded" by a possibility of choosing of strategies that are mutually harmful to both parties.

Bargaining procedures are strongly laden with the influence of competence of both players, their vulnerability or resistance to suggestion and stress. A general situation of the individuals also influences their attitudes, which may make them more or less concerned about time and agreement. Taking into consideration these and other factors which have not been named here, makes it difficult, if at all possible, to build a model for a bargaining situation. If anyway, we assume that the parties agree to appoint an objective arbiter who will show a just and profitable for both parties solution of a bargaining problem, the solution of this situation will mean building a scheme which should be used by the arbiter. There are many methods of indication of the cooperative solution. Two of the most eminent are Nash bargaining solution (1950, 1953) and Kalai-Smorodinsky bargaining scheme (1975). Both these methods, based on axiomatic approach were applied within this study.

Tariff rates negotiations are carried out by countries or integration groups. Tariff rates determine prices of imported goods, their quantity in international trade, profits for exporting companies and budget income of the importing country. Decisions about tariff rates are of great importance for the economy, even if we limit ourselves to the economic categories being under the direct influence of these decisions.

The purpose of this paper is to find an answer to the question, whether the chosen bargaining schemes can be applied for effective indication of cooperative solutions in negotiations on tariff rates. The achievement of the aim of this study was followed by the presentation of applied bargaining schemes and the formulation of the model producing benefit functions of both subjects of negotiations. Each of these two functions depends on two variables: tariff rates of bargaining countries. The article also presents the influence of the variability of the parameters of the model on the choice of cooperative solutions.

2. Bargaining solutions

A two-person bargaining situation was defined in a classic article by Nash as "the opportunity to collaborate for mutual benefit in more than one way. In the simpler case, no action taken by one of the individuals without the consent of the other can affect the well-being of the other one" (Nash, 1950a). As the examples of such actions, Nash gives: bilateral monopoly, duopoly, pay negotiation between employer and labour union and state trading between two nations. The latter will undergo a detailed analysis. But before that, let us take a closer look at the proposal by Nash.

The logic of the bargaining solution is based on a series of assumptions whose realization enables to construct a model of a bargaining situation. Firstly, one has to characterize the individuals participating in this situation. They are rational, their benefit functions are defined, are equal in bargaining skills and have a full knowledge of the choice parameters of the other. The second category of assumptions refers to the set of solutions (S) constituting benefit pairs for each partner, which can be achieved depending on the decisions made by them. According to Nash this set must be compact and convex (Nash, 1950a). The compactness causes that the set of solutions is bounded and can be closed in a suitably large square of Euclidean space. This implies that each continuous benefit function of one individual assumes the maximum value of the set for the given benefit of the other individual. The compactness of the set of solutions makes it possible, for a given solution, to find an alternative which increases benefit of at least one individual without decreasing the benefit of the other, only within one set. This can never happen by finding a solution which is the effect of a suitable mixture of strategies.

An important point in the set of solutions is the situation when there is a lack of cooperation between the individuals. The benefit functions at this point equal zero. Benefit increments reached by the individuals due to the cooperation will use this point as *status quo*. In other words *status quo* is a non-cooperative solution, to which the individuals may come back in case of bargaining failure. Setting zero coordinates of *status quo* is possible thanks to the assumption of the possibility of linear transformation of the set of solutions, which will not influence the choice of bargaining solution.

Nash claimed that in the set of solutions there is one solution which will bring each individual the benefit they expect. Thus, it can be assumed that rational individuals will agree to this or equivalent solution. There is a point which will be called a bargaining solution $N(S)$, belonging to the set of accessible solutions and treated by the individuals as mutually profitable. By finding the conditions characterizing the bargaining solution, Nash determined a simple method of assigning it.

The first condition is the consequence of the assumption of the individuals' rationality. If there exists a solution in the set S, such that there is another solution such as $u_1(\beta) > u_1(\alpha)$ and $u_2(\beta) > u_2(\alpha)$ then $\alpha \neq N(S)$ (u_i is the benefit function of subject i). This condition reflects the intention of the rational individuals to maximize the benefit within the frames of the agreed bargaining solution. Moreover, it limits the search for the bargaining solution to such subset S, in which all the points satisfy Pareto optimum criterion. In the geometrical sense the bargaining solution can be found in the right hand upper corner of the set S.

The second condition refers to the independence from the limitation of the set of solutions. If the set V includes the set S and $N(V)$ belongs to S, then $N(V) = N(S)$. If the bargaining solution is determined for the larger set V and belongs to the smaller set S, which is a sub set of V, then it is a bargaining solution for the smaller set. From all of the conditions, this one seems to be the most surprising. The others are a natural consequence of the accepted assumptions. Nash justified the use of this condition by claiming that if $N(S)$ is a bargaining solution for the larger set, then deleting from it certain solutions assented inaccessible (the set S is formed), does not lead to the change of indication.

The condition of independence from the limitation of the set of solutions raised most controversies. The instances to refute its justification are given by Straffin (Straffin, 2001, p. 136), Raiffa (Luce, Raiffa, 1964, pp. 125-131) and Kalai and Smorodinsky (Kalai, Smorodinsky, 1975). The critique of the conditions of Nash bargaining solution led the latter to formulate their own, alternative scheme, which will be presented later.

The third condition refers to the symmetry of the set S. If it includes a point with coordinates (a, b) then it also includes point (b, a). If S is symmetrical and functions u_1 and u_2 reflect its symmetry, point $N(S)$ has identical coordinates for each of the players (a, a). In other words, it lies on the line $u_1 = u_2$. This condition means the equal potential and bargaining skills of both individuals. In consequence, Nash bargaining solution is independent of the linear transformation of bargaining set.

Using the assumptions and three conditions based on them, Nash proved that the only criterion to determine point $N(S)$ is the maximization of the product $u_1 u_2$ in the first quarter in the system of coordinates, assuming the *status quo* in the zero point of Euclidean space. The compactness of the set S guarantees that such a point exists, its convexity makes it unique.

Nash developed his concept of bargaining solution in the second article about this problem (1953). He did not change the essence of the proposed solution. The new elements of the concept were setting the bargaining solution in the paradigm of game theory, he also gave them the form of axiom and specified *status quo* as the point of optimal threats.

The most essential supplement was the indication of the point of optimal threats as *status quo*. He was confident about this approach because he observed that a threat was an alternative solution in case negotiations failed. Each party can threaten that if negotiations fail they will use the threatening strategy. Players will formulate their threats in such a way that they could get the best *status quo* bearing in mind that the opponent will do the same. Choosing the threat strategy becomes thus a game similar to zero sum game, in which payoffs for the given pairs of strategies are subtractions of respective payoffs of the game, in which we look for a bargaining solution. Harm done to an opponent is benefit for the individual who does it. Optimal threat strategies are the solution of this game.

Applicability of Nash bargaining solution was presented with the analysis of quantity-variation duopoly from the standpoint of game theory (Mayberry, Nash, Shubik, 1953). The cooperative solution was determined, using the described scheme. In the first place, continuous payoff functions were determined for both individuals, referring to the chosen strategies (production and sales quantity). Each function depended on two variables: $\Pi_1(q_1, q_2)$ and $\Pi_2(q_1, q_2)$. To determine the optimal set in Pareto sense and optimal threats set one has to find pairs of strategies (q_1, q_2), for which Jacobi determinant equals zero:

$$|J| = \begin{vmatrix} \dfrac{\partial \Pi_1}{\partial q_1} & \dfrac{\partial \Pi_1}{\partial q_2} \\ \dfrac{\partial \Pi_2}{\partial q_1} & \dfrac{\partial \Pi_2}{\partial q_2} \end{vmatrix} = 0. \tag{1}$$

The next step was to determine a bargaining solution and an accompanying optimal threat point so that the sum of slopes of the tangent to the optimal set in Pareto sense at the point of bargaining solution and the line linking this point with its respective point of optimal threat, equals zero.

Nash's axiom on independence of irrelevant alternatives caused controversies among game theory scientists. There appeared many examples of games showing that, with the enlargement of the set of solutions S with the same *status quo* point, Nash bargaining solution disturbs the principle of fairness. Neglecting this principle makes it very difficult to reach the acceptance of both parties as to the particular cooperative solution. One of the most eminent examples of such criticism was the study of E. Kalai and M. Smorodinsky (1975).

They started from the extended Nash bargaining scheme (1953). The authors accepted three of his axioms: Pareto optimality of bargaining subset, symmetry of the set of solutions S and invariance with respect to affine transformation. They rejected the axiom of independence of irrelevant alternatives supporting this approach with the opinion of Luce and Raiffa (1964, s. 124).

Kalai and Smorodinsky substituted the rejected axiom with the axiom of monotonicity. The formulation of this axiom was preceded by the definition of the point $m(S)$ with coordinates:

$$u_{am}(S)=\sup\{u_a\in \mathrm{R}: \text{for any } u_b\in S\}, \tag{2}$$

$$u_{bm}(S)=\sup\{u_b\in \mathrm{R}: \text{for any } u_a\in S\}. \tag{3}$$

Coordinates of point $m(S)$ are the highest payoffs both players can obtain in the set of solutions S.

In a given bargaining pair that consists of status quo $T(S)$ and the set of solutions S, Kalai and Smorodinsky formulated their own axiom of monotonicity. It states that, if for every payoff available for player A, maximum payoff available for the player B rises, then the payoff within the cooperative solution attributed to the latter should also rise. Obviously, the axiom works analogically also in favor of player A.

Further, Kalai and Smorodinsky proved that there is only one bargaining scheme that fulfils all four axioms, presumed by the authors. It is the function that attributes to every bargaining pair ($T(S)$, S) the cooperative solution $KS(S)$ emerged as the intersection of Pareto optimal subset of the set S and the positively sloped diagonal of the smallest rectangle containing points $T(S)$ and $m(S)$.

The nature of both chosen bargaining schemes is shown on Figure 1. The set of solutions S is the polygon drawn with the continuous line. The Pareto optimal subset was indicated as the bolded side. Nash bargaining solution is located in the intersection of the line which joins it with the *status quo* and the Pareto optimal subset. It maximizes the product $(u_{aN} - u_{aT})(u_{bN} - u_{bT})$.The following condition must

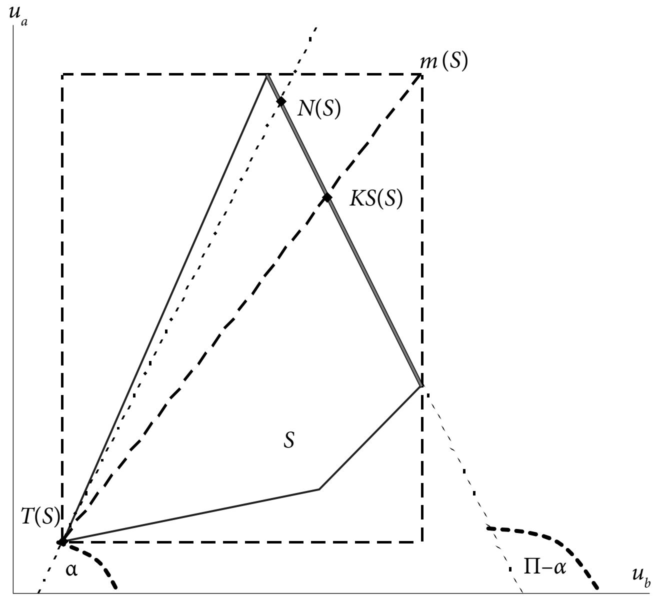

Figure 1. Comparison of cooperative solutions
Source: Own study based on (Nash, 1953) and (Kalai, Smorodinsky, 1975)

be satisfied: the slope of the Pareto optimal subset at point $N(S)$ plus the slope of the line joining $T(S)$ with $N(S)$ equal zero. The Kalai and Smorodinsky proposition emerges as the intersection of the Pareto optimal subset and the positively sloped diagonal of the smallest rectangle containing *status quo* and point $m(S)$.

3. Model presentation

The subject of the analysis is the situation of tariff rate bargaining between two countries A and B. To make it simpler it has been assumed that country A is an exclusive producer of goods whose only consumers live in country B. The quantity of these goods will be denoted by q_a. On the other hand, the goods produced exclusively by country B will be bought by customers in country A (q_b). This situation is nearly the same as double bilateral monopoly. Producers on the analyzed markets are so dispersed that they can be treated as perfectly competitive. Market equilibriums will set at the points of equal demand and supply.

Demand functions in countries A and B are expressed by respective equations:

$$p_a(q_b) = a - bq_b, \tag{4}$$

$$p_b(q_a) = a' - b'q_a, \tag{5}$$

where:
p_a – price of goods consumed in country A,
p_b – price of goods consumed in country B,
a, b, a', b' – positive constants.

Functions of aggregated supply of companies from countries A and B are respectively:

$$p_b(q_a)(1-D') = c' + d'q_a, \tag{6}$$

$$p_a(q_b)(1-D) = c + dq_b, \tag{7}$$

where:
D' – import tariff rate in country B taking values from interval < 0,1),
D – import tariff rate in country A taking values from interval < 0,1),
c, d, c', d' – positive constants.

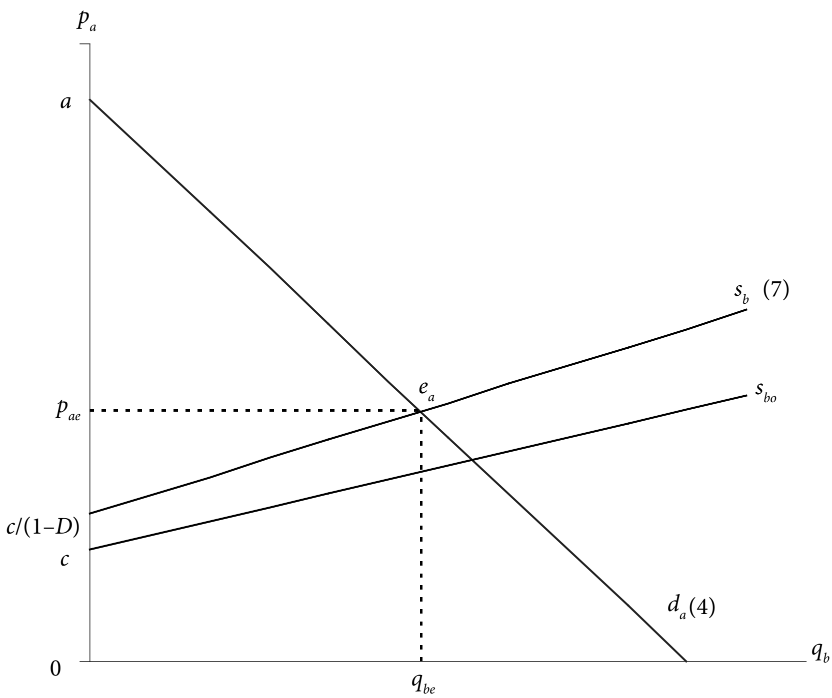

Figure 2. Demand in country A for the good produced in country B and its supply
Source: Own study

According to the assumptions, in country A, demand described by equation (4) meets the supply described by the equation (7). In country B demand described by equation (5) and supply from equation (6) decide about the equilibrium.

Figure 2 shows the situation in the market in country A. In order to indicate the influence of import tariff rates on the function, curve s_{bo}, which shows how the figure would look like at the zero tariff rate, was marked. Market equilibrium is set at point e_a with coordinates (q_{be}, p_{ae}). Numbers of equations are given next to demand and supply curves which are described by them.

The same situation is in the market in country B. It is depicted in Figure 3.

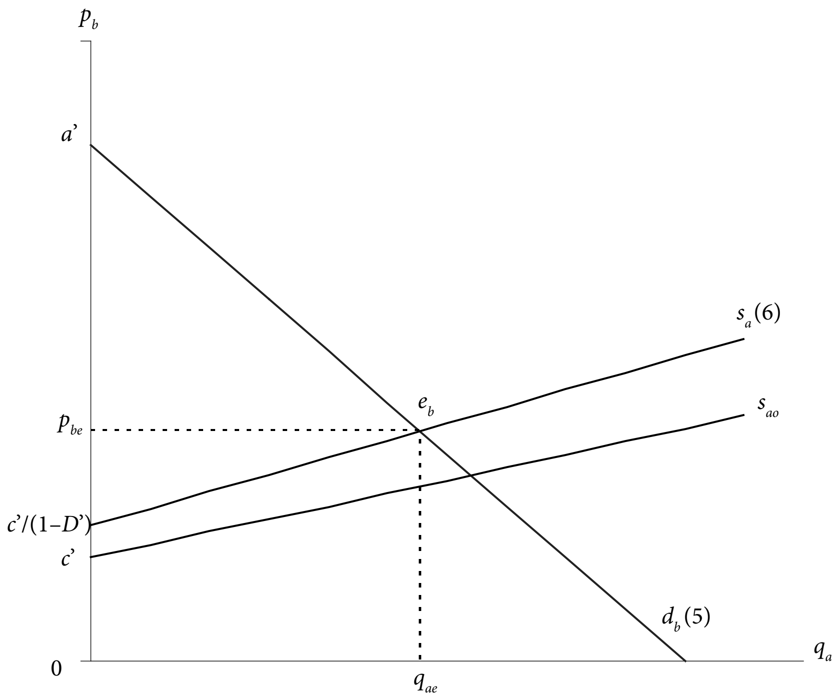

Figure 3. Demand in country B for the good produced in country A and its supply
Source: Own study

The character of the supply function, which takes into consideration the influence of tariff rates, causes that the quantity and price in equilibrium point on both markets are functions of the value of these rate. The forms of these functions are as follows:

$$q_{be}(D) = \frac{a(1-D)-c}{b(1-D)+d}, \tag{8}$$

$$p_{ae}(D) = a - b\frac{a(1-D)-c}{b(1-D)+d}, \tag{9}$$

$$q_{ae}(D') = \frac{a'(1-D')-c'}{b'(1-D')+d'}, \tag{10}$$

$$p_{be}(D') = a' - b'\frac{a'(1-D')-c'}{b'(1-D')+d'}. \tag{11}$$

The subject's benefits have been given a wide meaning while defining the benefits of both countries from the trade exchange. In consequence it comprises different components in terms of quality and subject. However, each of these components can be expressed in quantity form, depending on the level of country's own and partner's tariffs rates. The components of benefit function are: consumer's surplus realized on imported goods, producer's surplus realized by export companies and income from import customs duties.

Consumer's surplus has been set, according to its graphic interpretation, as the area between the demand curve and the equilibrium price in the goods quantity from zero to the equilibrium point (Blaug, 2000, p. 364). The area between points a, e_a and p_{ae} represents consumer's surplus in country A. Analogically, in country B the area of the triangle $\Delta a' e_b p_{be}$ will represent consumer's surplus.

Producer's surplus is a profit achieved by companies. If in both markets there is perfect competition, then the aggregated supply function is simultaneously the aggregated function of marginal cost of all the companies. The sum of their profits can be calculated as a definite integral over produced (and sold) quantity of the function which is the difference of equilibrium price and marginal cost in the interval from zero to equilibrium quantity:

$$\Pi_f(q_e) = \int_0^{q_e} \left(p_e - KM(q)\right) dq. \tag{12}$$

The area between the equilibrium price and demand curve in the interval up to the point of equilibrium will be a graphic representation of producer's surplus. Producers from country A will realize a surplus represented by the area of the triangle with the vertices at p_{bc}, e_b and $c'/(1 - D')$. In the case of producers from country B it will be the area limited by points p_{ae}, e_a and $c/(1 - D)$.

Income from customs duties will be a simple product of import value and a respective tariff rate. Benefit functions of both countries will have the following form (components are presented according to the order of appearance in the text):

$$\Pi_a(D, D') = \frac{1}{2}bq_{be}(D)^2 + \frac{1}{2}\left(p_{be}(D') - \frac{c'}{1-D'}\right)q_{ae}(D') + p_{ae}(D)q_{be}(D)D \tag{13}$$

$$\Pi_b(D,D') = \frac{1}{2}b' q_{ae}(D')^2 + \frac{1}{2}\left(p_{ae}(D) - \frac{c}{1-D} \right) q_{be}(D) + p_{be}(D') q_{ae}(D') D'. \quad (14)$$

Benefit functions for both countries depend on their own tariff rates and the rates of the partner country. The achieved benefit level depends not only on their own decisions but on their trade partner's, too. That creates a situation where decisions are made in conditions of uncertainty. "Such uncertainty appears when one or two actions have as an outcome, the set of certain possible results, in which, however, probabilities of these results are completely unknown or it does not make any sense to talk about probabilities"(Luce, Raiffa, 1964, p. 22). This is a situation when we can speak about a strategic game whose solution requires using the tools of game theory. One of them is determining a cooperative solutions according to the chosen bargaining schemes.

4. Determination of a cooperative solution

Indication of alternative cooperative solutions was performed on a model developed from the previous study of the author (due to be published in 2007). In a basic form, demand and supply functions are the same for both countries. Following versions will vary with respect of important parameters of the model. The unilateral change of parameters will affect the cooperative solutions. The consequences of $a > a'$, $c > c'$ and $d > d'$ will be examined.

Functions whose general forms were presented in equations (4–7) have been specified in the following way:

$$p_a(q_b) = 100 - 2q_b, \quad (15)$$

$$p_b(q_a) = 100 - 2q_a, \quad (16)$$

$$p_b(q_a)(1 - D') = 20 + 0.5q_a, \quad (17)$$

$$p_a(q_b)(1 - D) = 20 + 0.5q_b. \quad (18)$$

Substituting the chosen values of tariff rates to the assigned benefit functions (15–18) we get a matrix form of a strategic game in a symmetric version[1]:

[1] The range of variability was limited to 54%. First, with the tariffs higher than 80% equilibrium quantities were negative. Second, all the important values were within the taken range.

Table 1. Benefit values of country A

Πa(D,D’)		Tariff rate of country A (D)									
		0%	6%	12%	18%	24%	30%	36%	42%	48%	54%
Tariff rate of country B (D’)	0%	1280.0	1272.9	1261.5	1244.5	1219.9	1185.3	1137.0	1069.6	975.3	842.3
	6%	1301.0	1294.4	1283.7	1267.4	1243.8	1210.2	1163.0	1097.1	1004.6	873.9
	12%	1322.2	1316.3	1306.3	1290.9	1268.2	1235.8	1190.0	1125.6	1035.2	906.9
	18%	1343.5	1338.3	1329.1	1314.6	1293.0	1261.8	1217.5	1155.0	1066.7	941.3
	24%	1364.4	1359.9	1351.7	1338.2	1317.8	1288.0	1245.4	1184.9	1099.1	976.8
	30%	1384.1	1380.6	1373.3	1361.0	1342.0	1313.8	1273.1	1214.8	1131.9	1013.1
	36%	1401.7	1399.2	1393.1	1382.2	1364.7	1338.3	1299.7	1244.0	1164.2	1049.5
	42%	1415.6	1414.3	1409.5	1400.1	1384.3	1360.0	1323.9	1271.2	1195.0	1084.8
	48%	1423.4	1423.4	1420.2	1412.5	1398.8	1376.8	1343.5	1294.2	1222.3	1117.4
	54%	1421.1	1422.7	1421.3	1415.6	1404.3	1385.2	1355.1	1309.8	1242.8	1144.1

Source: Own study.

Table 2. Benefit values of country B

Πb(D,D’)		Tariff rate of country A (D)									
		0%	6%	12%	18%	24%	30%	36%	42%	48%	54%
Tariff rate of country B (D’)	0%	1280.0	1301.0	1322.2	1343.5	1364.4	1384.1	1401.7	1415.6	1423.4	1421.1
	6%	1272.9	1294.4	1316.3	1338.3	1359.9	1380.6	1399.2	1414.3	1423.4	1422.7
	12%	1261.5	1283.7	1306.3	1329.1	1351.7	1373.3	1393.1	1409.5	1420.2	1421.3
	18%	1244.5	1267.4	1290.9	1314.6	1338.2	1361.0	1382.2	1400.1	1412.5	1415.6
	24%	1219.9	1243.8	1268.2	1293.0	1317.8	1342.0	1364.7	1384.3	1398.8	1404.3
	30%	1185.3	1210.2	1235.8	1261.8	1288.0	1313.8	1338.3	1360.0	1376.8	1385.2
	36%	1137.0	1163.0	1190.0	1217.5	1245.4	1273.1	1299.7	1323.9	1343.5	1355.1
	42%	1069.6	1097.1	1125.6	1155.0	1184.9	1214.8	1244.0	1271.2	1294.2	1309.8
	48%	975.3	1004.6	1035.2	1066.7	1099.1	1131.9	1164.2	1195.0	1222.3	1242.8
	54%	842.3	873.9	906.9	941.3	976.8	1013.1	1049.5	1084.8	1117.4	1144.1

Source: Own study.

In the second version of the model equation (15) is changed to[2]:

$$p_a(q_b) = 120 - 2q_{ba}. \qquad (15')$$

In the third version equation (18) is changed to:

[2] All changes of parameters have the same relative value $\frac{\Delta a}{a} = \frac{\Delta c}{c} = \frac{\Delta d}{d} = 20\%$. Hence, the intensity of influence of changes can be compared.

$$p_a(q_b)(1-D) = 24 + 0.5q_b. \tag{18'}$$

Fourth version of the model includes equation (18) changed:

$$p_a(q_b)(1-D) = 20 + 0.6q_b. \tag{18''}$$

Non-cooperative Nash equilibrium of the game is D = D' = 28.57%. The choice of this strategy by both countries is a pair of strategies which are the best responses to each other. This pair of strategies is compliant with the equilibrium definition by Nash in nonzero games (Nash, 1950b). However, it is not an optimal solution in Pareto sense. Moving to the left, on the diagonal of the matrix we will find a solution which will increase benefits of both countries.

Benefit functions of both countries are continuous. Payoff matrices inform us only about their values for some pairs of arguments. Putting them on the figure can give an idea of the set of solutions on the plane. The graph informs us about its compactness, symmetry and convexity in the area of right hand, upper boundary representing an optimal subset in Pareto sense.

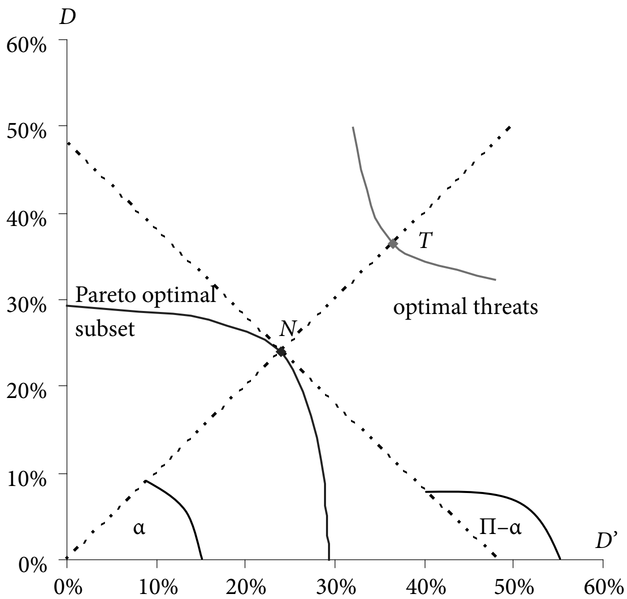

Figure 4. Cooperative solution according to Nash bargaining scheme
Source: Own study

According to the first condition of Nash bargaining solution, the cooperative solution will lie on the right hand upper boundary of the set of solutions. The symmetry of the set in the first version of the model enables us to use a simplified procedure which would consist in finding such tariff rates, for both countries, at which the product:

$$(\Pi_a(D_N, D'_N) - \Pi_a(D_T, D'_T))(\Pi_b(D_N, D'_N) - \Pi_b(D_T, D'_T)) = \max. \tag{19}$$

In order to present a universal procedure, its general formula has been applied.

The first step will be to equal Jacobi determinant for both benefit functions to zero

$$|J| = \begin{vmatrix} \dfrac{\partial \Pi_a}{\partial D} & \dfrac{\partial \Pi_a}{\partial D'} \\ \dfrac{\partial \Pi_b}{\partial D} & \dfrac{\partial \Pi_b}{\partial D'} \end{vmatrix} = 0. \tag{20}$$

This will let us single out an optimal subset in Pareto sense and subset of optimal threats from the set of solutions. However, before that we have to determine the form of derivatives necessary to use the equation (20). Their forms are as follows:

$$\frac{\partial \Pi_a}{\partial D} = bq_b(D)\frac{\partial q_b}{\partial D} + q_b(D)p_a(D) + Dq_b(D)\frac{\partial p_a}{\partial D} + Dp_a(D)\frac{\partial q_b}{\partial D}, \tag{21}$$

$$\frac{\partial \Pi_a}{\partial D'} = \frac{1}{2}\left(p_b(D') - \frac{c'}{1-D'}\right)\frac{\partial q_a}{\partial D'} + \frac{1}{2}q_a(D')\left(\frac{\partial p_b}{\partial D'} - \frac{c'}{(1-D')^2}\right), \tag{22}$$

$$\frac{\partial \Pi_b}{\partial D'} = b'q_a(D')\frac{\partial q_a}{\partial D'} + q_a(D')p_b(D') + D'q_a(D')\frac{\partial p_b}{\partial D'} + D'p_b(D')\frac{\partial q_a}{\partial D'}, \tag{23}$$

$$\frac{\partial \Pi_b}{\partial D} = \frac{1}{2}\left(p_a(D) - \frac{c}{1-D}\right)\frac{\partial q_b}{\partial D} + \frac{1}{2}q_b(D)\left(\frac{\partial p_a}{\partial D} - \frac{c}{(1-D)^2}\right). \tag{24}$$

The graphic picture of the set of combinations (*D, D'*) satisfying the equation (20) was presented in Figure 4. Contrary to Figure 5, the set of solutions was presented in the system of coordinates identical with the arguments of benefit function. Both approaches are accepted. In the first (Figure 4) the set of optimal threats is situated outside the set of optimal combinations in Pareto sense. In the second instance it is the other way round. The boundaries of Pareto optimal set in Figure 4 are set by responses to zero rate strategies chosen by the opponent, which satisfies equation (20) in a symmetric model (29.25%).

Point *N* denotes a cooperative solution determined according to Nash bargaining proposition. Point *T* denotes its respective optimal threat point. To assign them one has to use the fact that the slope of the tangent to the Pareto optimal set at point N and the line joining points N and T compensate to zero (Nash, 1953). If coordinates of points N and T are denoted respectively as (D_N, D'_N) and (D_T, D'_T), the slopes of suitable lines can be presented in the following way:

– the slope of the tangent can be expressed in two ways:

$$\mathrm{tg}(\Pi-\alpha)=\frac{\dfrac{\partial\Pi_b}{\partial D_N}}{\dfrac{\partial\Pi_a}{\partial D_N}}=\frac{\dfrac{\partial\Pi_b}{\partial D'_N}}{\dfrac{\partial\Pi_a}{\partial D'_N}}, \tag{25}$$

– the slope of the line joining points N and T:

$$\mathrm{tg}\alpha=\frac{\Pi_b(D_N,D'_N)-\Pi_b(D_T,D'_T)}{\Pi_a(D_N,D'_N)-\Pi_a(D_T,D'_T)}. \tag{26}$$

Finally we get a set of four equations with four variables D_N,D'_N, D_T and D'_T:

– equation (20) with variables (D_N, D'_N),
– equation (20) with variables (D_T, D'_T),
– equating the right side of (26) to the middle part of (25) multiplied by –1,
– equating the right side of (26) to the last part of (25) multiplied by –1.

Solving these four equations produces Nash bargaining solution (D_N, D'_N) accompanied by the pair of optimal threats (D_T, D'_T).

The point of optimal threats calculated within Nash bargaining solution was applied as the *status quo* in the Kalai and Smorodinsky method. The coordinates of optimal threats point in Figure 5 "cuts" from the upper-right boundary of the set of solutions the Pareto optimal subset. The limits of this subset are points (Π_{aT}, Π_{bm}) and (Π_{am}, Π_{bT}). Finding the Pareto optimal pairs of: $\Pi_a(D_T, D'_T)$ with its supplement Π_{bm} and $\Pi_b(D_T, D'_T)$ with its supplement Π_{am}, gives a pair of profits constituting coordinates of point m (Π_{am}, Π_{bm}). Kalai and Smorodinsky solution lies on the intersection of Pareto optimal set and the line mT.

Table 3 shows results of the research. Every analyzed version of the model brings both cooperative solutions. Either Nash bargaining scheme or Kalai and Smorodinsky can be applied to the model presented in this article.

First, symmetric version of the model brings the same cooperative solution regardless of an applied bargaining scheme. Bilaterally accepted tariff rate of 24.01% gives to the countries equal benefit of 1317.8. Optimal threats of both countries which "guard" the cooperative solutions, is the tariff rate 36.58%.

Growth of the demand in country A ($a > a'$) causes increase of the sum of benefits (Π_a+Π_b). Thanks to this positive change, benefits of both countries also increase. Strategic position of country A is now stronger than in the symmetric version. It

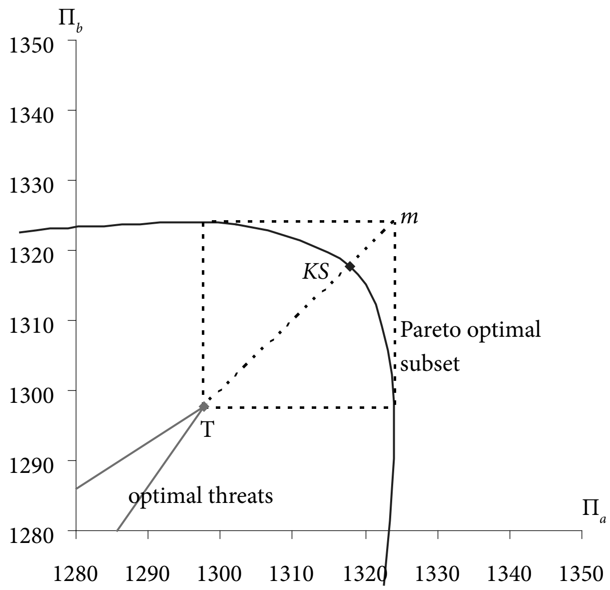

Figure 5. Cooperative solution according to Kalai & Smorodinsky proposition
Source: Own study

protects the country's own greater market with higher tariff rates according to both bargaining solutions. Obviously, market price in country A is higher. Simultaneously, price in country B almost does not change. The same observation emerges as far as the market equilibrium quantity is concerned. The tariff rate of country B in com-

Table 3. Cooperative solutions with optimal threat points in four versions of the model

Wariant		D (%)	D' (%)	$q_b(D)$	$q_a(D')$	$p_a(D)$	$p_b(D')$	$\Pi_a(D, D')$	$\Pi_b(D, D')$
$a = a'$ $b = b'$ $c = c'$ $d = d'$	T	36.58	36.58	24.6	24.6	50.9	50.9	1297.6	1297.6
	N	24.01	24.01	27.7	27.7	44.6	44.6	1317.8	1317.8
	KS	24.01	24.01	27.7	27.7	44.6	44.6	1317.8	1317.8
$a > a'$ $b = b'$ $c = c'$ $d = d'$	T	38.78	35.49	31.0	24.9	58.0	50.3	1898.1	1454.3
	N	27.90	24.39	34.3	27.6	51.5	44.7	1917.9	1472.1
	KS	28.33	23.74	34.1	27.8	51.7	44.4	1918.7	1471.3
$a = a'$ $b = b'$ $c > c'$ $d = d'$	T	33.07	38.44	23.3	24.0	53.3	52.0	1190.8	1188.8
	N	20.51	23.45	26.6	27.8	46.9	44.3	1213.6	1286.3
	KS	19.76	24.05	26.7	27.7	46.6	44.6	1212.6	1287.5
$a = a'$ $b = b'$ $c = c'$ $d > d'$	T	37.61	39.39	22.9	23.7	54.1	53.0	1225.3	1306.7
	N	24.94	23.19	26.2	27.9	47.6	44.2	1251.0	1338.8
	KS	24.23	23.81	26.4	27.8	47.3	44.5	1250.0	1340.1

Source: Own study.

parison to the symmetric version almost does not change (proposition N slightly rises D', method KS slightly lowers it). Nevertheless, benefit of country B increases significantly, thanks to the enlargement of the market on which firms from B operate. Benefits of country A increase much more intensely.

Parameter c or c' is the ordinate of aggregate marginal cost for the zero quantity. It is the minimum price level demanded by producers to supply the product. The lower the parameter c, the higher the technological potential of the country is. It can simply produce cheaper. The third version of the model ($c > c'$) shows the consequences of competitive advantage of country B. The sum of benefits (Π_a+Π_b) lowers in comparison to the symmetric model. This change is transferred to the individual benefits, but in the case of country A it diminishes more significantly. Market price, quantity and the tariff rate do not change in country B in comparison to the first version, especially in KS cooperative solution (in Nash bargaining there are small differences). Country A, which produces its product at a higher cost, suffers greater decrease of benefits. It is accompanied by the higher market price, lower quantity and a significantly diminished tariff rate. Reduction of tariff, in country A stimulates import, which increases consumer surplus and budget income from duty tariffs. These benefits substitute a significant decrease of profits experienced by the firms from country A.

Parameter d or d' is the slope of the aggregate marginal cost of producers. The steeper the supply curve is, the faster the marginal cost grows. For smaller quantities the difference is not so significant, but when quantity increases the gap between marginal costs of the countries will be getting larger. The fourth version of the model with $d > d'$ brings reduction of the aggregate benefits (Π_a+Π_b) in comparison to the symmetric option, but not as significant as in the third one. There is another difference in comparison to $c > c'$ version. Benefit of country B is higher than in the symmetric option. Again, market price and equilibrium quantity hardly change in country B. In country A the reactions of $q_b(D)$ and $p_a(D)$ have the same directions as in the third version, but are a little bit more intense. Tariff rate in country B slightly falls and in country A slightly rises. Average of D and D' is almost the same as in the symmetric version.

There is one interesting feature of the comparison of bargaining solutions. In every asymmetric version of the model the gap between benefits is larger within Kalai and Smorodinsky proposition. It exploits more intensely strategic advantage of the country. In the author's previous unpublished study where quantity variation duopoly was used, the observation was just the opposite. Nash bargaining solution differed profits more intensely. It may be concluded that the nature of bargaining solutions strongly depends on the nature of the model in which they are applied.

5. Summary

The purpose of the article was to show the possibility of using chosen bargaining solutions to find the cooperative solution in a specific bargaining situation. The situation concerned determining tariff rates in trade exchange between two countries. Based on the mechanism of a simple model, benefit functions were built for both parties. These functions depended on the players' own strategy and the strategy of the negotiating partners. The result was a game in which tariff rates became the strategies, countries participating in trade exchange were the players, and the payoffs were benefit values for specific pairs of strategies. It was a nonzero game. It happens very often that in the games of this type, the solution, which consists in finding Nash equilibrium, is not optimal in Pareto sense. Both players can benefit from an agreeable choice of a solution which meets this criterion and satisfies the sense of justice for both parties. Nash bargaining solution and Kalai and Smorodinsky proposition are a formal tool to indicate such a course of action.

Calculations, which were made, confirmed that using both solution lets us indicate the cooperative solution in a chosen bargaining situation. Their outcome confirmed hypothetical indication based on the analysis of payoffs matrices. In addition, the application of Nash bargaining solution enabled the author to determine pairs of strategies of optimal threats, which can be used in case the players give up the cooperative solution. Application of chosen bargaining solutions brought the conclusion that the outcome of indications of cooperative solutions strongly depends on the nature of the explored economic model.

The examination of influence parameters' changes proved that worsening the situation of the subject led to decrease of its benefit in every case. In one case it also caused the decrease of benefit of the other party.

Achievinh the purpose of the article creates a perspective for further research. It would be interesting to show how the cooperative solution would change if there was a non perfect competition in both markets.

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