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The modeling of technology diffusion – theory and empirics

Abstract: The aim of the article is to review the classical approach to the modeling of technology diffusion and to introduce two modifications of this approach. We present the setup of these modifications and the results of the empirical analysis based upon them. We discuss the role of technology diffusion in a group of 28 OECD countries in the period 1981–1999.
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1. Introduction

Modern theory of economic growth, whose milestones were Lucas' (1988) and Romer's models (1990), stresses the role of knowledge accumulation in the process of economic growth. It is commonly assumed that knowledge is embodied in human capital and technology. The difference between the two "carriers" of knowledge lies in the fact that while technology is a non-rival good (if someone uses knowledge, it does not mean that someone else cannot use it either), the use of human capital is exclusive, i.e. its application in one sort of activity makes it impossible to use it in some other form of activity at the same time. Thus, the average levels of human capital in various countries are independent of one another. The situation with technology is somewhat different. Non-rivalry of technology makes it possible to use some technological solutions from one country in other countries at the same time. This means that the levels of technology in different countries can be interrelated. The reason for this is the process of technology transfer (technology diffusion)[2].

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[2] For a review of technology diffusion in aggregate models of economic growth in Polish see e.g. Cichy, Malaga (2007). For a review in English, see e.g. Benhabib, Spiegel (2005).

Such a process is an empirical fact. Many technological solutions that were created in one country are nowadays used in most countries of the world. A good example of such a solution is the computer – invented in the United States in the 1940s and popularized there in the 1980s, is now commonly used virtually everywhere and produced in many countries of the world, including those that did not take part in the development of technology of production of computers. This means that there has been a transfer of a technological solution (how to produce a computer) from the United States to e.g. China or Taiwan. Poland is one of the leading European producers of cars. However, the technologies of production of all the types of cars produced in Poland have been invented in other countries and they have been transferred to Poland. This means that foreign investments are an important source of technology diffusion. The most advanced technological solutions are devised in the leading (from the technological point of view) countries and then they are used in other countries of the world (especially the ones with cheap workforce), increasing the efficiency of production in these countries. This is the crux of the phenomenon of technology transfer.

The issue of measuring the level of technology in different countries is not straightforward in itself. There is no universal agreement among researchers on how to understand and measure technology. In this paper we understand the level of technology of a given country operationally – i.e. it is a variable of the production function associated with the efficiency of combining the factors of production in a given way. If we say e.g. that the level of technology of Poland is lower than that of the USA, it means that the production obtained from a given combination of physical capital, labour and human capital (the standard factors of production in neoclassical growth theory) will be higher in the USA than in Poland. Such operational approach to technology is widely used in the economic growth theory and it is regarded now as the standard approach.

The process of technology diffusion can be observed in the time series of technology indicators. For example, if one analyzes the rate of technological progress (which is operationally defined as the rate of growth of the technological variable) and R&D expenditures in different countries, one can notice that the countries whose expenditures are relatively small can have relatively high rates of technology growth, indicating that many technological solutions come from other countries and are not a result of their own R&D activity.

However, an important question is: what is the role of the process of technology diffusion in technological progress and economic growth? Which countries benefit the most from technological solutions invented elsewhere? What is the most effective way to model such processes?

This paper tries to give an answer to these questions. The main idea is to define a quantity called the technology diffusion coefficient and find its value for different countries. A high value of the coefficient will mean that technology transfer to the country under consideration is effective, whereas a low value will suggest low efficiency of technology diffusion.

The paper is organized as follows. In Sec. 2 we review the approach of Nelson and Phelps (1966) and Benhabib and Spiegel (2005). In Sec. 3 and 4 we analyze modifications of the Nelson-Phelps and Romer's models. Sec 5. contains concluding remarks.

2. Nelson-Phelps' and Benhabib-Spiegel's models of technology diffusion

2.1 Nelson-Phelps' approach

Nelson and Phelps (1966) assumed that the level of technology of a given country i, which they denote by $A_i(t)$, can increase due to two effects. First, it can be a result of the country's own research activity, i.e. devoting resources that could be used elsewhere to purposeful activities aimed at inventing new technological solutions. Second, the growth of the level of technology can also result from introducing technological solutions invented in some other country, called the technological leader.

Nelson and Phelps proposed the following equation to describe the influence of these two processes:

$$\frac{\dot{A}_i(t)}{A_i(t)} = g\left(h_i(t)\right) + c\left(h_i(t)\right)\left(\frac{T(t)}{A_i(t)} - 1\right), \tag{1}$$

where $g(h_i(t))$ denotes an increasing function of human capital $h_i(t)$, describing technology growth as an effect of the country's own R&D activity (the first effect), $c(h_i(t))$ is an increasing function of human capital, describing the efficiency of technology transfer (the second effect) from the leader country, whose technology level is $T(t)$.

The intuition behind equation (1) is the following. The larger the resource of human capital in a given country and the more technologically underdeveloped the country is, the higher the rate of technological progress. The differential equation (1) can be solved analytically, if we assume that the level of human capital is constant over time. If we denote by g_i, c_i (which can now be called the diffusion coefficient), and g the values of the functions $g(h_i(t))$, $c(h_i(t))$ and $g(h(t))$, respectively, for constant human capital resources, the solution of (1) takes the form:

$$A_i(t) = \left(A_i(0) - \Omega_i T(0)\right) e^{(g_i - c_i)t} + \Omega_i T(0) e^{gt}, \tag{2}$$

where $\Omega_i \equiv c_i/(c_i - g_i + g)$.

One can also show that:

$$\lim_{t\to\infty}\frac{A_i(t)}{T(t)}=\Omega. \tag{3}$$

Regardless of the parameter values, in the limit $t \to \infty$ the rate of technological progress in all countries is constant and the relative levels of technology do not change over time any more. If $g_i = g$, the i-th country's technology will catch up with the leader's technology.

2.2 Benhabib and Spiegel's approach

Benhabib and Spiegel (2005) proposed a modification of the Nelson-Phelps' model. They added an additional factor in the diffusion term of the technology dynamics equation:

$$\frac{\dot{A}_i(t)}{A_i(t)}=g\left(h_i(t)\right)+c\left(h_i(t)\right)\frac{A_i(t)}{T(t)}\left(\frac{T(t)}{A_i(t)}-1\right). \tag{4}$$

The consequence of adding this factor is that when the technological backwardness of a given country is very high, the rate of technology diffusion is very low, which reflects the difficulties of implementing too distant technologies.

If the levels of human capital are constant over time, the analytical solution to the differential equation (4) is given by:

$$A_i(t)=\frac{A_i(0)e^{(g_i+c_i)t}}{1+\frac{A_i(0)}{T(0)}\frac{c_i}{c_i+g_i-g_m}\left(e^{(c_i+g_i-g_m)t}-1\right)}. \tag{5}$$

One can easily show that:

$$\lim_{t\to\infty}\frac{A_i(t)}{T(t)}=\begin{cases}(c_i+g_i-g)/c_i & \text{if} \quad c_i+g_i-g>0\\ A_i(0)/T(0) & \text{if} \quad c_i+g_i-g=0,\\ 0 & \text{if} \quad c_i+g_i-g<0\end{cases} \tag{6}$$

i.e. that the relative level of technology of a given country in the limit $t \to \infty$ depends on the values of the parameters. Technological convergence is possible only if the combined effect of technology diffusion and R&D activity (c_i+g_i) is larger than the effect of R&D activity in the leading country (g). Otherwise, the distance in technology levels can stay at the initial level or increase.

The consequence of such formulation is that there is some threshold value of the level of human capital in a given country that makes technological convergence possible. Thus, technological divergence that is empirically observed for some countries (Benhabib and Spiegel (2005)) does not have to be permanent and can turn into convergence due to investments in human capital.

The fact that technological divergence is possible in some countries suggests that Benhabib and Spiegel's model is more realistic than the original formulation of Nelson and Phelps.

3. Modification of the Nelson-Phelps' approach

3.1. Setup of the model

Nelson and Phelps' approach to the modeling of the R&D term in the technology dynamics equation is in many ways oversimplified, e.g. for the technological leader, one assumes that the rate of technological progress is exogenously given.

Moreover, the argumentation of Jones (1996, 2005) shows that assuming the derivative of the level of technology with respect to time to be linear in the technology level is contradictory to empirical data. Therefore, Jones supported the idea of assuming a more general form of the technology dynamics equation. The problem with such an approach is, however, that the assumption of non-linearity e.g. in Romer's model leads to non-balanced growth. The fact that the concept of balanced growth does not apply to the analysis of technology diffusion anyway makes it natural to introduce non-linear effects in this case.

One can do this by assuming the following form of technology dynamics equation:

$$\dot{A}_i(t) = \xi_i(t)A_i(t)^{1-\gamma} + d_i(t)\left(\frac{T(t)}{A_i(t)} - 1\right)A_i(t), \tag{7}$$

where $\xi_i(t)$ is a measure of expenditures on R&D activity (it corresponds to the function $g(h_i(t))$ in the original Nelson-Phelps' formulation), $d_i(t)$ denotes technology diffusion coefficient in the *i*-th country (it corresponds to the function $c(h_i(t))$), γ – a parameter which measures the scale effects of R&D activity (the same for all countries).

When $\gamma \equiv 0$, we recover the original Nelson-Phelps' model.

Equation (7) has a particularly useful solution, easily found analytically, if we assume that in the period under analysis the factor $T(t)/A_i(t)$-1 and the variables $\xi_i(t)$ and $d_i(t)$ are constant. This suggests than we can analyze the model empirically if we restrict ourselves to short periods of time, i.e. such that the assumption that the respective variables are constant is plausible. Then, the solution of (7) is:

$$A_i(t)=\left(\left(A_i(\tau)^{\gamma}-\frac{\xi_i(\tau)}{d_i(\tau)\Lambda_i(\tau)}\right)e^{-\gamma d_i(\tau)\Lambda_i(\tau)(t-\tau)}+\frac{\xi_i(\tau)}{d_i(\tau)\Lambda_i(\tau)}\right)^{1/\gamma}, \qquad (8)$$

where $\Lambda_i(\tau) \equiv T(\tau)/A_i(\tau)$-1. The solution (8) is meaningful only in the range $t \in [\tau, \tau + \tau_{max}]$, where $\tau + \tau_{max}$ denotes the moment, for which the assumption of constancy of the respective variables is not plausible any more.

We will assume $\tau_{max} = 1$ year and proceed recursively. First, we will take $A_i(\tau)$, $\Lambda_i(\tau)$, $\xi_i(\tau)$ and $d_i(\tau)$ at $\tau = 0$ and calculate $A_i(t)$ for τ between 0 and 1. Then, we will take $A_i(\tau)$, $\Lambda_i(\tau)$, $\xi_i(\tau)$ and $d_i(\tau)$ at $\tau = 1$ and find $A_i(t)$ for τ between 1 and 2. We will proceed in a similar manner for all the moments of interest.

3.2. Empirical analysis

To examine the conclusions that result from the modified technology dynamics equation (7) and its solution (8), we have performed an empirical test for a 28-country subgroup of OECD[3] during 1981-1999.

We assume that $A_i(0)$ is equal to its empirical value[4] for 1981 relative to the value for the United States, which is assumed to be the technological leader and the source of the frontier technologies.

We take $\xi_i(\tau)$ to be the R&D expenditures as a fraction of GDP p.c. (OECD (2005)).

The parameter γ has been calibrated to match the model and empirical dynamics of technology as closely as possible for the United States. As the matching criterion we choose the following quantity (mean relative error – MRE):

$$MRE_i=\sum_{t=1}^{18}\frac{\left|A_i(t)-A_i^{emp}(t)\right|}{A_i^{emp}(t)}, \qquad (9)$$

where $A_i(t)$ is the technology level, which results from the model at time t, and $A_i^{emp}(t)>0$ is the empirical value at this time.

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3 As of today, OECD has 30 members. We do not consider Slovakia and Luxembourg here, because the former joined OECD after the period of analysis and for the latter the relevant statistical data are unavailable.

4 The empirical values have been calculated from the data regarding the levels of GDP p.c. and physical capital p.c. of the OECD countries. The exact method of this calculation has been described in the Ph.D. thesis of Cichy (2007) (in Polish).

The calibrated value of γ is 0.96. However, we will also analyze the predictions of the model for other reasonable values of this parameter[5]: 5/4, 1, 3/4, 1/2, 1/4, 0, and -1.

The diffusion coefficients for every country, calibrated for every country to match the model and empirical dynamics, have been gathered in Table 1. The higher the value of the diffusion coefficient, the more effective the process of technology transfer to a given country. The values of *d* close to 0 mean that the effects of technology

Table 1. Technology diffusion coefficients

Country	The values of d with γ equal to:							
	1.25	1.00	0.96	0.75	0.50	0.25	0.00	−1.00
AUS	0.012	0.014	0.014	0.015	0.017	0.019	0.020	0.025
AUT	0.014	0.017	0.017	0.019	0.021	0.023	0.024	0.025
BEL	0.018	0.018	0.018	0.018	0.017	0.017	0.017	0.015
CAN	0.019	0.020	0.020	0.021	0.023	0.024	0.025	0.027
CZE	0.011	0.013	0.013	0.014	0.015	0.016	0.017	0.018
DEN	0.024	0.026	0.027	0.028	0.030	0.031	0.033	0.034
FIN	0.018	0.021	0.021	0.023	0.025	0.027	0.028	0.030
FRA	−0.012	−0.009	−0.009	−0.007	−0.004	−0.002	0.001	0.008
GER	−0.017	−0.013	−0.012	−0.010	−0.006	−0.003	0.000	0.010
GRE	0.015	0.016	0.016	0.016	0.017	0.017	0.017	0.018
HUN	−0.016	−0.014	−0.013	−0.012	−0.010	−0.009	−0.008	−0.005
IRL	0.043	0.045	0.045	0.046	0.048	0.049	0.051	0.052
ISL	0.066	0.066	0.066	0.066	0.065	0.064	0.063	0.056
ITA	0.042	0.044	0.044	0.045	0.047	0.048	0.048	0.048
JPN	−0.007	−0.001	0.000	0.004	0.008	0.012	0.015	0.024
KOR	0.003	0.008	0.009	0.012	0.015	0.018	0.020	0.022
MEX	0.006	0.007	0.007	0.008	0.008	0.008	0.008	0.008
NED	−0.006	−0.004	−0.003	−0.001	0.001	0.004	0.006	0.011
NZL	0.006	0.008	0.008	0.009	0.011	0.012	0.013	0.016
NOR	0.041	0.043	0.044	0.044	0.045	0.045	0.046	0.048
POL	−0.002	0.001	0.001	0.002	0.004	0.004	0.005	0.006
POR	0.015	0.016	0.017	0.017	0.018	0.019	0.019	0.020
SPA	0.014	0.015	0.015	0.016	0.017	0.018	0.019	0.021
SWE	−0.036	−0.033	−0.033	−0.030	−0.027	−0.024	−0.022	−0.012
SWI	−0.045	−0.044	−0.044	−0.043	−0.042	−0.040	−0.039	−0.033
TUR	−0.001	0.001	0.001	0.002	0.003	0.003	0.004	0.004
UK	−0.013	−0.010	−0.010	−0.007	−0.005	−0.002	0.000	0.008

Source: own calculations.

[5] The values of the parameter γ have been chosen arbitrarily and they correspond to various (reasonable) degrees of non-linearity in the technology dynamics equation (7).

diffusion are negligible for the country under consideration and negative values of the diffusion coefficient suggest that technology transfer from a given country is stronger than technology transfer to this country.

The values of the technology diffusion coefficients differ significantly for the countries under analysis. The biggest fluctuations are observed for the highest values of the parameter γ – from -0.045 to 0.066. For the negative value of γ, the fluctuations are much smaller.

As we have said before, the levels of the diffusion coefficients give the information about the strength of the diffusion processes in a given country. We can infer that technology diffusion was very effective in Ireland, Iceland, Italy and Norway. The role of technology diffusion was the smallest in France, Germany, Hungary, Sweden and Switzerland.

For the latter group of countries, we have observed that the diffusion coefficients are negative (in the case of France and Germany only for $\gamma > 0$). When we analyze the time series of expenditures on R&D (excluding Hungary), we can notice that the level of expenditures is close to the American or even higher than in the United States. At the same time, growth rates of technology are approximately equal to the American growth rate, despite the fact that they should be higher, because the level of technology is in these countries lower than in the United States. The negative values of technology diffusion coefficients seem to suggest that the efficiency of allocation of the R&D expenditures must be lower than in the USA.

Therefore, a desirable way to develop the model further would be to introduce a measure of efficiency of R&D expenditures. Introducing such a variable $\varepsilon_i(t)$, we could write the technology dynamics equation as:

$$\dot{A}_i(t) = \varepsilon_i(t)\xi_i(t)A_i(t)^{1-\gamma} + d_i(t)\left(\frac{T(t)}{A_i(t)} - 1\right)A_i(t). \tag{10}$$

A possible measure of the efficiency of R&D expenditures could be academic and patent-based. It seems to be theoretically justifiable to assume that the average number of publications per e.g. 1000 researchers is high in a country which allocates its R&D resources effectively, and low if the R&D expenditures are allocated in an ineffective way. Similarly, a high number of patents per 1000 researchers suggests that R&D resources are allocated effectively, whereas if this number is low, some of the R&D expenditures are allocated ineffectively.

A good measure of the efficiency of R&D activity could be human capital based. One can suppose that if we consider two countries with the same amount of resources allocated for R&D, the country with a larger amount of human capital will use them up more effectively. This could result in a bigger number of patents and publications per researcher, so this measure would to some extent be consistent with the measures described earlier.

Another question regarding human capital is whether the diffusion coefficient depends on human capital resources of a given country. We have calculated average levels of human capital for the group of countries under analysis with the use of the Manuelli-Seshadri model (2006)[6].

The key feature of the Manuelli-Seshadri model is that it takes into account not only the average length of the schooling process in a given country but also the quality of this process. This makes the calculated values of human capital more realistic.

Figure 1 shows the pairs (human capital, technology diffusion coefficient) ≡ (h, d) for the OECD countries under analysis.

The plot clearly suggests that the Nelson-Phelps' hypothesis should be rejected. According to the model we consider, there is no dependence of the technology diffusion coefficients on the levels of human capital of the countries under analysis. An interesting question arises here, whether adding a measure of R&D activity efficiency can alter this picture in a significant way. The hypothesis of Nelson and Phelps

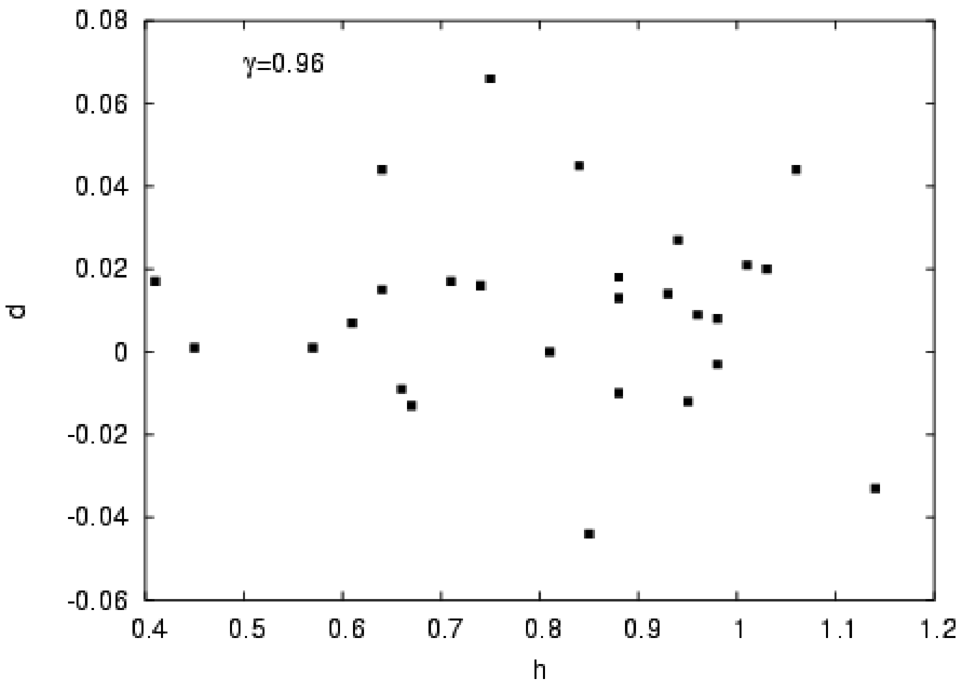

Figure 1. Human capital levels and technology diffusion coefficients for the group of OECD countries under analysis
Source: own calculations

[6] A description of the model and its application to the analysis of human capital in Poland and in the USA can be found e.g. in Cichy, Malaga (2006). The application to the group of OECD countries can also be found in the doctoral thesis of Cichy (2007) (in Polish).

is very elegant and we can suppose that if all the relevant factors in the analysis are taken into account, it may turn out to be true.

To conclude this section, let us discuss the average level of the diffusion coefficient. Depending on how we interpret the negative levels of this coefficient, the calculated average is between 1 and 2 percent. This value means that in a country whose technology is twice lower than in the United States, the rate of technology growth will be 1 to 2 percentage points higher than in the USA, assuming the same level of R&D expenditures. Thus, we have isolated the diffusion effect and the obtained value seems to be realistic.

4. Endogenous model of technology with technology diffusion

4.1. Setup of the model

The model that we analyze in this section is an extension of the Romer's model (1990) of endogenous technological change.

Let us consider an economy which consists of three sectors: production, intermediate goods and research sector. Let the production function take the following form:

$$Y(t) = \left(B(t)H_Y(t)\right)^{1-\theta} \int_0^{\infty} x(i,t)^{\theta}\, di, \tag{11}$$

where $x(i,t)$ denotes the amount of the i-th intermediate good used in production at time t, $H_Y(t)$ – the amount of human capital used in the production sector at this time, and $B(t)$ – the effectiveness of implementation of the technology of intermediate goods production.

Following Romer, let us define physical capital $K(t)$ as:

$$K(t) = \int_0^{A(t)} x(i,t)\, di = A(t)\bar{x}(t), \tag{12}$$

where $A(t)$ denotes the range of intermediate goods that can be used in production at time t. By symmetry (all goods are equally important), it follows that $\forall i \le A(t),\ x(i,t) = \bar{x}(t)$.

The production function can then be written as:

$$Y(t) = K(t)^{\theta} \left(A(t)B(t)H_Y(t)\right)^{1-\theta}. \tag{13}$$

This is the Cobb-Douglas production function with Harrod-neutral technology, being the product of two functions – $A(t)$ and $B(t)$. A and B are complementary technological factors. To develop the country's technology, it is necessary to invest in both of these factors.

Thus, we actually have two "research sectors", whose role is to raise the level of the product $A(t)B(t)$. The marginal rate of substitution between A and B equals $B(t)/A(t)$, which means that one unit of A (which can be interpreted as a technology of production of one intermediate good) can be replaced by $B(t)/A(t)$ units of B (i.e. by raising the level of implementation of intermediate goods production technology by $B(t)/A(t)$). In case the ratio of technological factors is far from optimal (i.e. one of the factors dominates), its further development brings relatively less gains than raising the level of the other factor. Therefore, it is not desirable to either only introduce new technologies, or only implement old technologies – the optimal choice is to find the optimal combination of the two.

Let us now assume that the intermediate goods production technologies can originate only from outside of a given country, i.e. the growth of $A(t)$ can only result from a techno-logy diffusion effect. To meet this assumption, let us take the following form of the dynamics equation for $A(t)$:

$$\dot{A}(t) = d\frac{h_A(t)}{\tilde{h}_{AB}}\left(\frac{T(t)}{A(t)} - 1\right)A(t), \tag{14}$$

where d is the diffusion coefficient, $h_A(t)$ the amount of human capital p.c. used in the diffusion sector, $\tilde{h}_{AB}$ is the total amount of human capital p.c., which is used in both research sectors in the last period of analysis, and $T(t)$ is the range of intermediate goods available in the technological leader country.

Taking equation (14), we assume that the technological leader renders the technologies of intermediate goods production accessible to other countries the more willingly, the older (the more distant from the technology frontier) the technology is. The larger the stock of "research" human capital devoted to introducing new technologies, the faster the technology transfer. We also assume that the growth rate of technology is linear in the stock of human capital.

The variable $B(t)$ will be interpreted as the level of adjustment of intermediate goods production technologies to the characteristics of the economy of a given country (e.g. the economy's structure). We assume the following form of the $B(t)$ dynamics equation:

$$\dot{B}(t) = \xi(t)\left(\frac{h_B(t)}{\tilde{h}_{AB}}\right)^{\eta} B(t)^{1-\gamma}, \tag{15}$$

where $\xi(t)$ denotes the expenditures on technology implementation, $h_B(t)$ the amount of human capital in the implementation sector, and γ and η are parameters.

We denote by $h_{AB}(t)$ the sum of $h_A(t)$ and $h_B(t)$ and we assume that it is equal to $h(t)$- $h_Y(t)$, i.e. the difference of the total human capital resources of the economy and the resources used in production.

We also assume that $\eta \in (0, 1)$ $\eta \in (0,1)$, i.e. there are decreasing returns to scale in the implementation sector, since the work of some researches can lead to the same effects.

The dynamics of technology in the technological leader country is governed by the same type of equation as (15):

$$\dot{T}(t) = \xi(t)\left(\frac{h_T(t)}{\tilde{h}_T}\right)^{\eta} T(t)^{1-\gamma}, \tag{16}$$

where $\tilde{h}_T$ denotes the resource of human capital devoted to research in the last period of analysis, and $h_T(t)$ the amount of human capital used in the research sector at time t.

The solutions to the differential equations (14)-(16) take the form:

$$A(t) = \left(A(0) - T(0)\right) e^{-d\frac{h_A(0)}{\tilde{h}_{AB}}t} + T(0), \tag{17}$$

$$B(t) = \left(B(0)^{\gamma} + \gamma\xi(0)\left(\frac{h_B(0)}{\tilde{h}_{AB}}\right)^{\eta} t\right)^{1/\gamma}, \tag{18}$$

$$T(t) = \left(T(0)^{\gamma} + \gamma\xi(0)\left(\frac{h_T(0)}{\tilde{h}_{AB}}\right)^{\eta} t\right)^{1/\gamma}. \tag{19}$$

By optimal allocation of human capital between research sectors we will understand such allocation that the growth rate of production p.c. reaches its maximum. If we denote the optimal level of human capital in the diffusion sector by $h_A^*(t)$ and in the implementation sector by $h_B^*(t)$, one can easily show that the two quantities fulfill the following equations:

$$h_B^*(t) = \tilde{h}_{AB}\left(\frac{d}{\eta}\frac{B(t)^{\gamma}}{\xi(t)}\left(\frac{T(t)}{A(t)} - 1\right)\right)^{\frac{1}{\eta-1}}, \tag{20}$$

$$h_A^*(t) = h_{AB}(t) - \tilde{h}_{AB}\left(\frac{d}{\eta}\frac{B(t)^{\gamma}}{\xi(t)}\left(\frac{T(t)}{A(t)} - 1\right)\right)^{\frac{1}{\eta-1}}. \tag{21}$$

4.2. Empirical analysis

The empirical analysis of the model under consideration has been performed in a similar manner to the analysis of the modification of the Nelson-Phelps' approach. We apply the model to the same group of 28 OECD countries in the period 1981-1999.

As $\xi(t)$ we again use the R&D expenditures as a fraction of GDP p.c. (OECD (2005)). To find the amount of human capital that is used in the research sectors, we use the data on the number of researchers per 1000 employed (OECD (2005)). We assume that an average researcher has a three times[7] larger amount of human capital than an average inhabitant of the respective country. The total amount of human capital for 1999 has been found from the Manuelli-Seshadri model. We assume that in the preceding years it decreases at an average rate of 1% per year.

To find the levels of technological variables $A(t)$ and $B(t)$ in 1981, we assume that the following relationship with the empirical value $A^{emp}(0)$ holds[8]:

$$A(0) = B(0) = \sqrt{A^{emp}(0)}. \tag{22}$$

We will consider the following values of the parameter η: 0.01, 0.19[9], 0.25, 0.5, 0.75, 0.99. The parameter γ has been calibrated in such a way that the model and empirical dynamics for the technological leader were as close as possible, with the mean relative error as the matching criterion. For the values of the parameter η under consideration, the calibrated values of γ are: 0.94 (for η=0.01), 0.61 (0.19), 0.49 (0.25), 0.24 (0.50), -0.21 (0.75) and -0.64 (0.99).

For other countries of interest, we have calibrated the technology diffusion coefficients to match the empirical and model technology dynamics, with the matching criterion defined as:

$$MRE_i = \sum_{t=1}^{18} \frac{\left|A_i(t)B_i(t) - A_i^{emp}(t)\right|}{A_i^{emp}(t)}. \tag{23}$$

Table 2 shows the calibrated values of the technology diffusion coefficient for the group of countries under analysis. The values that we have found are again considerably varied and much higher than for the model from Sec. 3. To make them comparable to this model, we can define an effective diffusion coefficient:

7 The case of other ratios of human capital of a researcher to human capital of an average inhabitant has also been investigated and it leads to very similar conclusions.

8 This relationship is the "first guess". It would be interesting to investigate other possible relationships between *A* and *B* in the future.

9 The value 0.19 corresponds to a minimal mean relative error of GDP p.c. estimation based on the Manuelli-Seshadi model – see Cichy (2007).

$$d_{eff}(t) = d\frac{h_A(t)}{\tilde{h}_{AB}}. \tag{24}$$

The countries in which the diffusion sector dominates have the effective diffusion coefficient approximately equal to the diffusion coefficient. For other countries, there can be significant differences in the values of the two quantities.

Table 2. Technology diffusion coefficients for the endogenous model of technology with technology diffusion

Country		The values of d with:					
	$\gamma=$	−0.64	−0.21	0.24	0.49	0.61	0.94
	$\eta=$	0.99	0.75	0.50	0.25	0.19	0.01
AUS		0.217	0.206	0.168	0.111	0.096	0.049
AUT		0.415	0.391	0.318	0.210	0.183	0.055
BEL		0.222	0.206	0.152	0.091	0.078	0.035
CAN		0.444	0.397	0.237	0.126	0.108	0.051
CZE		0.088	0.085	0.076	0.058	0.053	0.030
DEN		0.527	0.491	0.397	0.258	0.217	0.099
FIN		0.687	0.667	0.621	0.360	0.276	0.107
FRA		0.334	0.292	0.081	0	0	0
GER		0.216	0.154	0.066	0.027	0.020	0
GRE		0.169	0.165	0.161	0.150	0.145	0.119
HUN		0	0	0	0	0	0
IRL		0.445	0.438	0.432	0.412	0.386	0.301
ISL		0.999	0.875	0.606	0.328	0.281	0.153
ITA		0.334	0.318	0.261	0.186	0.163	0.091
JPN		0.346	0.324	0.252	0.115	0.079	0.020
KOR		0.098	0.096	0.089	0.072	0.065	0.038
MEX		0.029	0.029	0.029	0.027	0.027	0.023
NED		0.176	0.148	0.089	0.045	0.037	0.010
NZL		0.128	0.124	0.109	0.068	0.060	0.031
NOR		0.587	0.584	0.444	0.319	0.244	0.115
POL		0.024	0.023	0.023	0.019	0.018	0.011
POR		0.210	0.206	0.200	0.184	0.181	0.155
SPA		0.162	0.159	0.153	0.137	0.130	0.085
SWE		0.256	0.181	0	0	0	0
SWI		0.380	0.097	0	0	0	0
TUR		0.020	0.019	0.019	0.017	0.016	0.011
UK		0.191	0.150	0.059	0.023	0.017	0

Source: own calculations.

In the model under consideration we can observe values of the diffusion coefficient equal to 0. Such value means that decreasing d meant an ever-increasing matching between the model and empirical technology dynamics. However, negative values of d are forbidden, since $d = 0$ means that the whole human capital that can be devoted to research is used in the implementation sector ($h_{AB} = h_B$). Similarly to the model from Sec. 3, if we take into account differences in the efficiency of R&D activities, we can expect that for some countries it would be optimal to employ human capital in the diffusion sector as well. In such case, the values of the diffusion coefficient would be positive.

For the highest values of the parameter η (which means the lowest values of the parameter γ), the diffusion coefficients are the highest and they are positive for almost all countries. Decreasing η and increasing γ means that the growth rate of the variable $B(t)$ is higher, so that the implementation sector becomes more efficient relatively to the diffusion sector. The effect of the above is that the values of the diffusion coefficients decrease.

It is worth mentioning that the values of the technology diffusion coefficients in Table 2 are the highest for the countries for which they were the highest in the model from Sec.3 – Ireland, Iceland and Norway, and additionally for Denmark and Finland, for which they were also relatively high in that model.

The values "0" are obtained for the countries for which they were negative in the previous model – France, Norway, Hungary, Sweden and Switzerland, and additionally the United Kingdom. The convergence of the results of application of the two models allows us to draw a conclusion that the role of technology diffusion is indeed relatively small.

Figures 2-4 show three types of allocation of human capital between the research sectors and the dynamics of the effective diffusion coefficient for Norway, Sweden and Germany ($\eta = 0.19$, $\gamma = 0.61$). Figures 5-7 show the model and empirical technology dynamics for these countries.

The first group of countries, represented by Norway on the plots, consists of the ones in which the diffusion sector dominates. Typically, around 90% of human capital was used in the diffusion and only 10% in the implementation sector. The effective diffusion coefficient was relatively high. The consequence of such allocation is that the variable $A(t)$ grows much faster than $B(t)$. Besides Norway, this group of countries consists of Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, Greece, Ireland, Iceland, Italy, Japan, South Korea, Mexico, New Zealand, Poland, Portugal, Spain, and Turkey. Among these countries one can observe subgroups in which the ratio $h_A(t)/h_B(t)$ was approximately constant (Austria, Denmark, Finland, Greece, Ireland, Iceland, Norway, Portugal, Spain), decreased (South Korea, Mexico, Italy) or increased, with constant $h_B(t)$ (Australia, Poland, Turkey), or with decreasing $h_B(t)$ (Belgium, Canada, the Czech Republic, Japan, New Zealand).

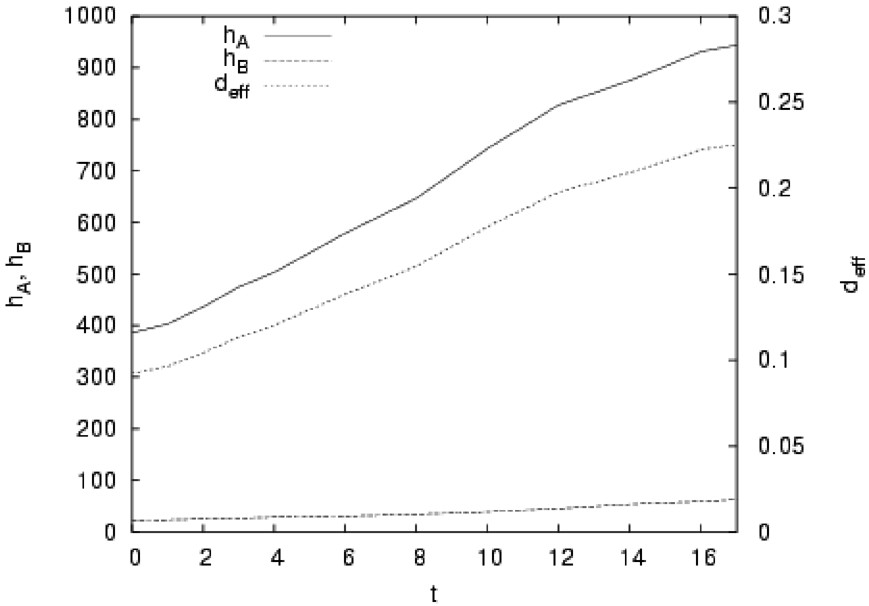

Figure 2. The allocation of human capital in the research sectors and the effective technology diffusion coefficient for Norway

Source: own calculations

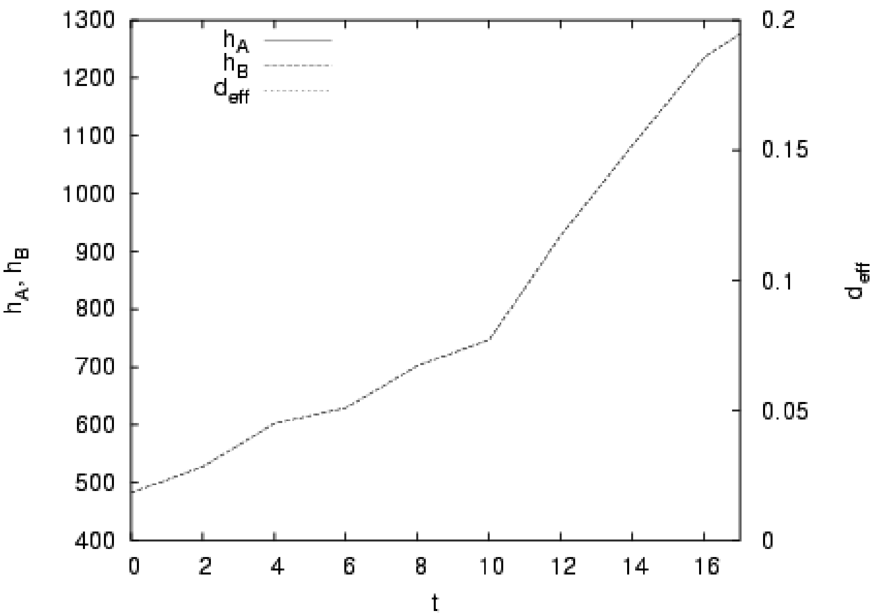

Figure 3. The allocation of human capital in the research sectors and the effective technology diffusion coefficient for Sweden

Source: own calculations

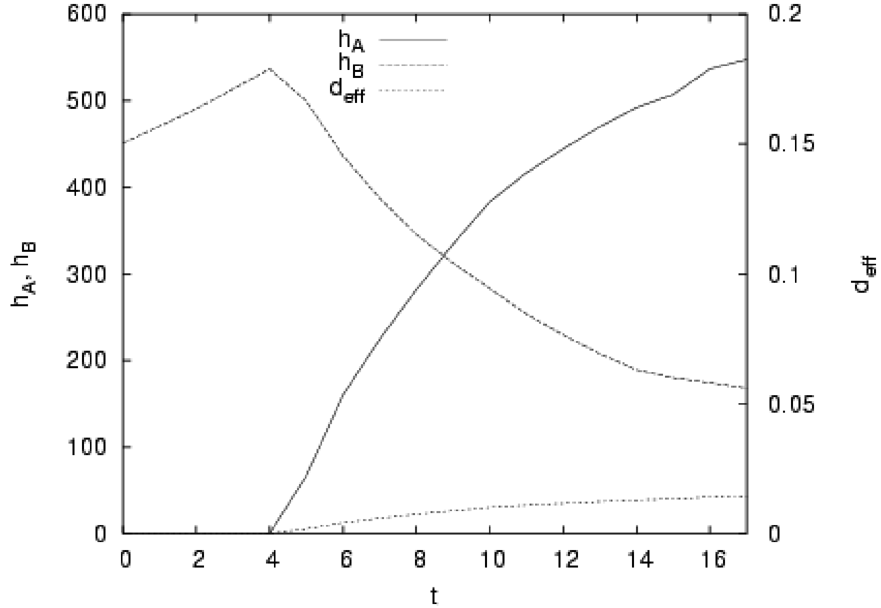

Figure 4. The allocation of human capital in the research sectors and the effective technology diffusion coefficient for Germany
Source: own calculations

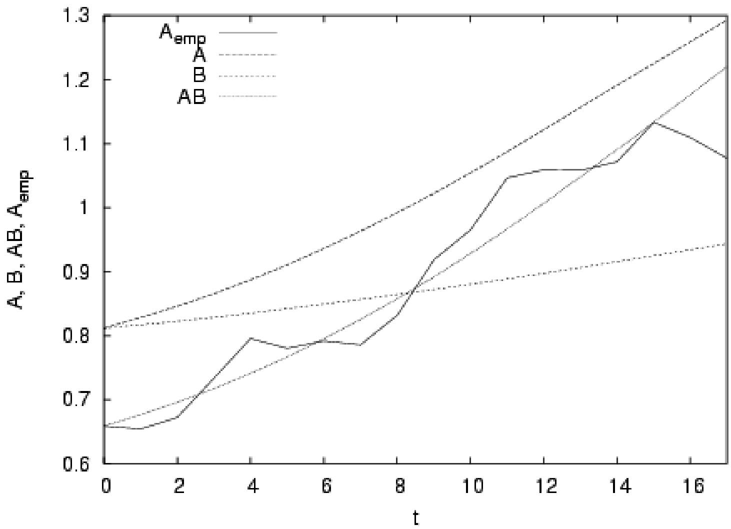

Figure 5. The model and empirical technology dynamics for Norway
Source: own calculations

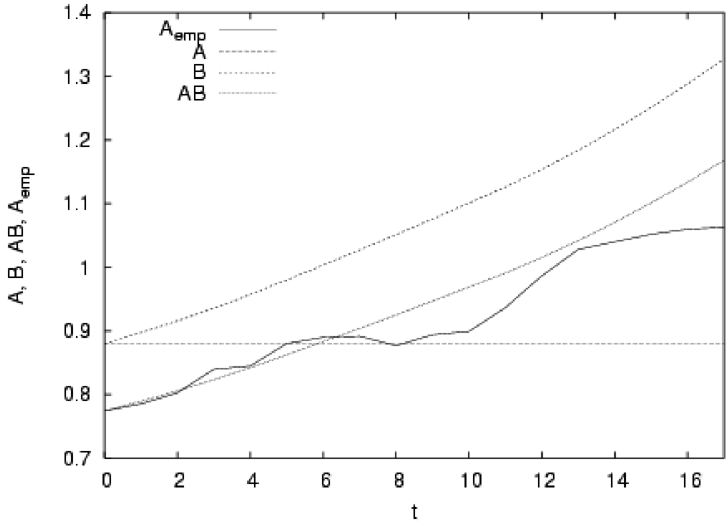

Figure 6. The model and empirical technology dynamics for Sweden
Source: own calculations

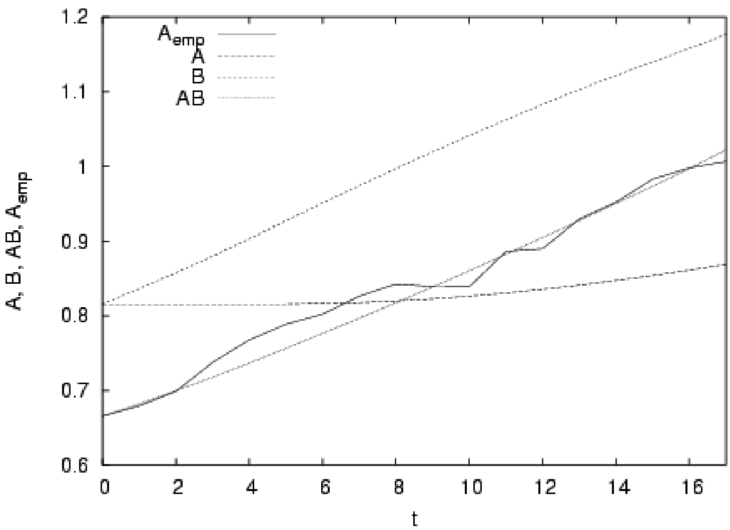

Figure 7. The model and empirical technology dynamics for Germany
Source: own calculations

The second group of countries consists of France, Hungary, Sweden (on the plots) and Switzerland. The diffusion coefficient for these countries was zero, which means that the whole resource of research human capital was employed in the implementation sector. Therefore, $A(t)$ was constant over the period of analysis and the level of technology growth was due to the growth in the level of the variable $B(t)$.

The third group of countries is represented on the plots by Germany and consists also of the Netherlands and the United Kingdom. For these countries we have observed something intermediate between the cases of the countries from the first and the second group. In the beginning, the implementation sector dominated and there was no human capital allocated in the diffusion sector. Then, around 1985, there was a constant flow of human capital from the implementation to the diffusion sector which started to dominate around the year 1990. At the end of the period of analysis, around 75-90% (depending on the country) of human capital was employed in the diffusion sector.

To conclude, let us mention the limitations introduced by the type of modeling that has been presented in this section. First, like in the model analyzed in Sec. 3, this model does not take into account the differences in the efficiency of R&D activities in various countries. Second, we had to introduce some arbitrary assumptions in the empirical analysis, notably the one that at the beginning of the period of analysis the two technological variables were equal, and that the η and γ parameters were the same in all countries.

5. Conclusion

In this paper we have presented the classical approach to the modeling of technology diffusion and two modifications thereof.

The first one was to introduce a more complex form of the R&D term in the technology dynamics equation, following Jones's discussion of the scale effects in research and development.

The second was an endogenous model of technological progress with technology diffusion. The level of technology of a given country was expressed as a product of two variables, representing technology diffusion and implementation.

For both models we have performed an empirical analysis and we have found that the conclusions that can be drawn from both of them are convergent. We have distinguished a group of countries in which the diffusion processes were especially strong, and a group in which they had relatively small effects. The first group consists of Ireland, Iceland, Italy, Norway, Denmark and Finland. The role of technology diffusion was the smallest in France, Germany, Hungary, Sweden, Switzerland and the United Kingdom (the second group).

The principal conclusions of the models and the whole diffusion-based approach to the modeling of technological progress seem to be interesting and worth further exploring. Especially, introducing the efficiency of R&D activities and technology transfer between multiple countries (not only between a given country and the technological leader) would be interesting.

However, even now we can claim that technology transfer is undoubtedly a vital mechanism of technological progress, and consequently, of economic growth.

References

Benhabib J., Spiegel M. (2005), *Human Capital and Technology Diffusion*, in: *Handbook of Economic Growth*, ed. P. Aghion, S. Durlauf, North-Holland.

Cichy K., Malaga K. (2006), *The Quantity and Quality of Human Capital in Poland and USA Analyzed in the Framework of the Manuelli-Seshadri Model*, PUE Review, Vol. 6, No. 1, pp. 5–24.

Cichy K., Malaga K. (2007), *Dyfuzja technologii w zagregowanych modelach wzrostu gospodarczego*, proceedings of the conference 'Modelowanie i prognozowanie gospodarki narodowej', Jastrzębia Góra 28–31.05, under revision.

Cichy K. (2007), *Kapitał ludzki i postęp techniczny jako determinanty wzrostu gospodarczego w nowej teorii wzrostu*, Ph.D. thesis, Poznań University of Economics, Department of Economics.

Jones C. (1996), *Human Capital, Ideas, and Economic Growth*, VIII Villa Mondragone International Economic Seminar on Finance, Research, Education, and Growth, Roma 25–27.06.

Jones C. (2005), *Growth and Ideas* in: *Handbook of Economic Growth*, ed. P. Aghion, S. Durlauf, North-Holland.

Lucas R. (1988), *On the Mechanics of Economic Development*, Journal of Monetary Economics, No. 22, pp. 3–42.

Manuelli R., Seshadri A. (2006), *Human Capital and the Wealth of Nations*, Working Paper, http://www.econ.wisc.edu/~aseshadr/working_pdf/humancapital.pdf.

Nelson R., Phelps E. (1966), *Investment in Humans, Technological Diffusion, and Economic Growth*, American Economic Review No. 56, pp. 69–75.

OECD Factbook 2005, http://caliban.sourceoecd.org/vl=362938/cl=20/nw=1/rpsv/fact2005/06-01-01-t01.xls.

Romer P. (1990), *Endogenous Technological Change*, Journal of Political Economy No. 98, pp. S71–S102.