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A TEST ON A PRODUCTION NETWORK-BASED RBC MODEL

Abstract: What is the origin of aggregate fluctuations represents one of the important questions in macroeconomics and constitutes a challenge to macroeconomic modelling. Battiston, Delli Gatti, Gallegati, Greenwald, & Stiglitz (2007) present an intuitive model which, relying on firms' bankruptcy and rebirth as a source of business cycle dynamics, is able to replicate stylized facts of industrial demography. Although firms' exit behaviour constitutes the core of their framework, the empirical investigation in terms of ability to match the business exit patterns observed in the data is still unexplored. This study contributes to the existing literature by examining if the firms' exit patterns observed in the model of Battiston et al. (2007) can replicate those observed in U.S. business statistics. The results show that the model is able to partially match firms' exit patterns in the data, specifically the counter-cyclicality of exit rate and the co-movement between the aggregate output growth and the average size of exiting firms. On the other hand, the results of a calibrated model are counterfactual in terms of a strong negative contemporaneous correlation between the average size of exiting firms and output growth.

JEL classification: E32, D85, L14, L22.

Keywords: supply chain; bankruptcy; business cycle; agent based modelling.

WERYFIKACJA EMPIRYCZNA MODELU RBC BAZUJĄCEGO NA POŁĄCZENIACH W RAMACH SIECI PRODUKCYJNEJ

Streszczenie: Źródło fluktuacji gospodarczych stanowi jedno z głównych niewyjaśnionych zjawisk w makroekonomii. Battiston, Delli Gatti, Gallegati, Greenwald i Stiglitz (2007) zaprezentowali w swojej pracy model sieci produkcyjnej pozwalający na odtworzenie wybranych faktów stylizowanych z zakresu demografii przedsiębiorstw, a przy tym prowadzący do powstania dynamiki o charakterze cyklicznym w skali ogółu podmiotów. Kluczowym elementem modelu prowadzącym do uzyskania wskazanych wyników jest przy tym schemat wychodzenia przedsiębiorstw z rynku. Pomimo istotnej roli tego aspektu weryfikacja empiryczna rezultatów generowanych przez model nie znalazła się w obszarze bezpośredniego zainteresowania autorów. Celem niniejszego artykułu jest kalibracja wskazanego modelu do danych, a następnie ocena, czy schemat opuszczania rynku przez przedsiębiorstwa wynikający z modelu, który zaprezentowali Battiston i in. (2007), odpowiada wzorcom obserwowanym w danych z sektora przedsiębiorstw w Stanach Zjednoczonych. Z przeprowadzonej analizy wynika, że wyniki ze wskazanego modelu są tylko w ograniczonym stopniu zbieżne z danymi, m.in. w zakresie antycykliczności stopy wychodzenia przedsiębiorstw z rynku (ang. *exit rate*) oraz zależności pomiędzy tempem wzrostu zagregowanego produktu a przeciętną wielkością firmy opuszczającej rynek.

Słowa kluczowe: łańcuch dostaw; bankructwo przedsiębiorstwa, cykl koniunkturalny, modelowanie wieloagentowe.

Introduction

The origin of aggregate fluctuations represents one of the important puzzles in macroeconomics, constituting a challenge to macroeconomic modelling. The issue has so far been addressed from different angles (Rebelo, 2005). In real business cycles (RBC) models, as introduced by Kydland & Prescott (1982), exogenous technological shocks have been used to generate the cyclicality of aggregate variables. Prescott (1986) even refers to economic fluctuations as constituting the optimal reaction to the variation of technological change. This approach has been criticized by Romer (2016), who criticized the RBC and dynamic stochastic general equilibrium (DSGE) models widely used in recent decades for “attributing fluctuations in aggregate variables to imaginary causal forces that are not influenced by the action that any person takes”.

Alternative ways of dealing with the puzzle include assuming the importance of oil shocks, fiscal shocks, investment-specific technical change or monetary shocks (Rebelo, 2005). The importance of oil price dynamics on aggregate fluctuations has been suggested by i.a. Rasche & Tatom (1981), Gisser & Goodwin (1986) and Carruth, Hooker, & Oswald (1998). Kim & Loungani (1992) formally verify the extent to which energy price shocks are able to replace technological shocks as a source of business cycle fluctuations. They conclude, however, that the reliance of the model on the technology shocks is only modestly reduced after the energy price shocks are included and therefore the latter shocks shall be treated only as a supplementary element. Concerning the second group of shocks, Christiano & Eichenbaum (1992) as well as Baxter & King (1993) represent examples of studies that stress that taking note of fiscal policy helps in bringing the results of RBC models closer to the data. Another approach suggests using investment-specific technical change as an alternative to the technological shock present in a standard real business cycle framework. While in the latter case the shock affects all factors of production by making them more productive, the former one applies only to new capital goods. Fisher (2002) claims that in the U.S. investment-specific shocks have accounted for about 50% of the business cycle variation in hours of work, with the standard neutral technological shock having accounted for as little as 6% of the variable variation. Moreover, according to his study, investment-specific shocks are able to explain around 40% of output variation. The importance of investment-specific shocks in shaping the business fluctuations has also been confirmed by i.a. Greenwood, Hercowitz, & Krusell (2000). Finally, monetary shocks have been in the center of attention for numerous authors using the RBC modelling framework; see Christiano, Eichenbaum, & Evans (1999) for an extensive review of the literature.

In the case of all of the abovementioned approaches the microeconomic shocks affecting particular agents have no role in a macroeconomic fluctuation, since they average out in aggregation, thus shutting down endogenous dynamics that may arise from the interactions between particular agents in the economy. Stiglitz (2018) suggests that "at the heart of the failure [of DSGE models] were the wrong micro-foundations, which failed to incorporate key aspects of economic behaviour, e.g. incorporating insights from information economics and behavioural economics". A recent important contribution has also been made by Acemoglu, Carvalho, Ozdaglar, & Tahbaz-Salehi (2012), who showed that in the presence of the input-output network between different sectors of the economy shocks at a disaggregated level (affecting particular sectors) do not necessarily cancel out in the aggregate.[1] This mechanism undermines the seminal argument of Lucas (1977), who claimed that the aggregate effect of micro-level shocks is negligible due to the fact that these shocks tend to average out. If the microeconomic shocks indeed play a role in shaping the aggregate outcome, they need to be duly taken into account at the modelling stage.

The 2008 global financial crisis made it even more evident that the structure of connections among interacting agents can greatly influence the stability of the network as a whole and the behaviour of particular agents in the future. Even the failure of a relatively small bank had a systemic implication, causing a serious risk of the domino effect.

While the interbank market seems to have represented one of the most evident and popular fields of research for network based economics in recent years[2] it is definitely not the only field where network structure matters. Interactions between companies constituting a production network are another relevant example. So far the agent based modelling approach has been applied to the field of production networks by i.a. Bak, Chen, Scheinkman, & Woodford (1993) and Weisbuch & Battiston (2007). Battiston et al. (2007) present an intuitive model that – on the one hand – is based on relatively simple assumptions, yet – on the other hand – this parsimonious structure is sufficient to replicate stylized facts of business demography.[3] The model relies on firms' bankruptcy and rebirth as a source of economic fluctuations. Although firms' exit behaviour constitutes the core of the framework proposed by Battistion et al. (2007), its ability to match the business exit patterns observed in the data is still not examined.

The aim of this study (and its contribution to the existing literature) is to compare a firm's exit patterns observed in a calibrated model in Battiston et al. (2007) with the corresponding ones in the U.S. data.

The rest of the paper is structured as follows. Firstly, the next section contains information about the data used for the purpose of this study. Secondly, the model of Battiston et al. (2007) is described, followed by details

[1] See also Acemoglu, Akcigit, & Kerr (2015) and Acemoglu, Carvalho, Ozdaglar, & Tahbaz-Salehi (2016) for the follow-up research on this topic.

[2] See e.g. Allen & Gale (2000); Upper & Worms (2004); van Lelyveld & Liedorp (2006); Wells (2004); Nier, Yang, Yorulmazer, & Alentorn (2007); Boss, Elsinger, Summer, & Thurner (2004); Glasserman & Young (2013); Hałaj & Kok (2013).

[3] The model is based on relatively simple assumptions, i.a. in terms of its structure and dynamics. For a summary of the current knowledge with regard to these issues see e.g. Csermely, London, Wu, & Uzzi (2013).

on the calibration procedure. Finally, the results section provides information about the model's ability to track the dynamics observed in the U.S. data, concluded by a discussion of directions in which the current framework could be further developed.

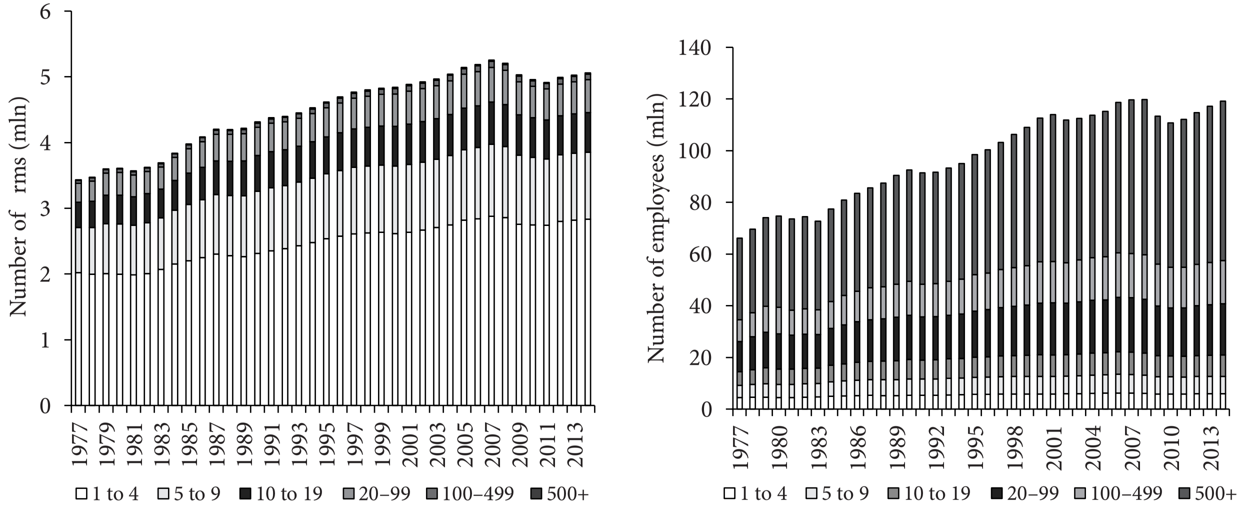
1. Data

The data used for the purpose of this study comes from the Business Dynamics Statistics (BDS) database provided by the U.S. Census Bureau. The time series are of yearly frequency, ranging from 1977 to 2014.[4] BDS is a publicly available compilation from the Longitudinal Business Database and provides detailed information about firms'/establishments' entry and exit as well as employment creation and destruction. The coverage and scope of the time series follows the approach of the Census Bureau's County Business Patterns (CBP) program with self–employed individuals, employees of private households, railroad employees, agricultural production employees, and most government employees constituting the major groups being excluded.[5] Although data are available at the level of firms and establishments, in this study only the former have been considered, as the latter are not suitable for tracking the business exit phenomena. While a database provides information on business dynamics by firm size (both initial and current), firm age, sector, U.S. state, metro/non-metro, in this study the emphasis has been placed on the first two dimensions (age and size). The U.S. Bureau of Economic Analysis has been used as a source for annual U.S. GDP data.

The total number of companies in the U.S. has been steadily increasing, with the exception of the years 1981, 1988 and 2009–2011 when a decline was observed (see Figure 1, left). The distribution of companies according to a firm's size shows the domination of very small entities – firms which employ up to 19 workers account for 88% of the total on average. As confirmed by Axtell (2001), the entire firm size distribution in the U.S. can be described by the Pareto distribution. However, the distribution changes considerably when the total number of employees in each group is analysed (see Figure 1, right). The largest fraction of employee works in large com-

[4] In case of the selected variables data for 2015 are also available, yet for consistency reasons they were not used in this study.

[5] https://www.census.gov/ces/dataproducts/bds/methodology.html.



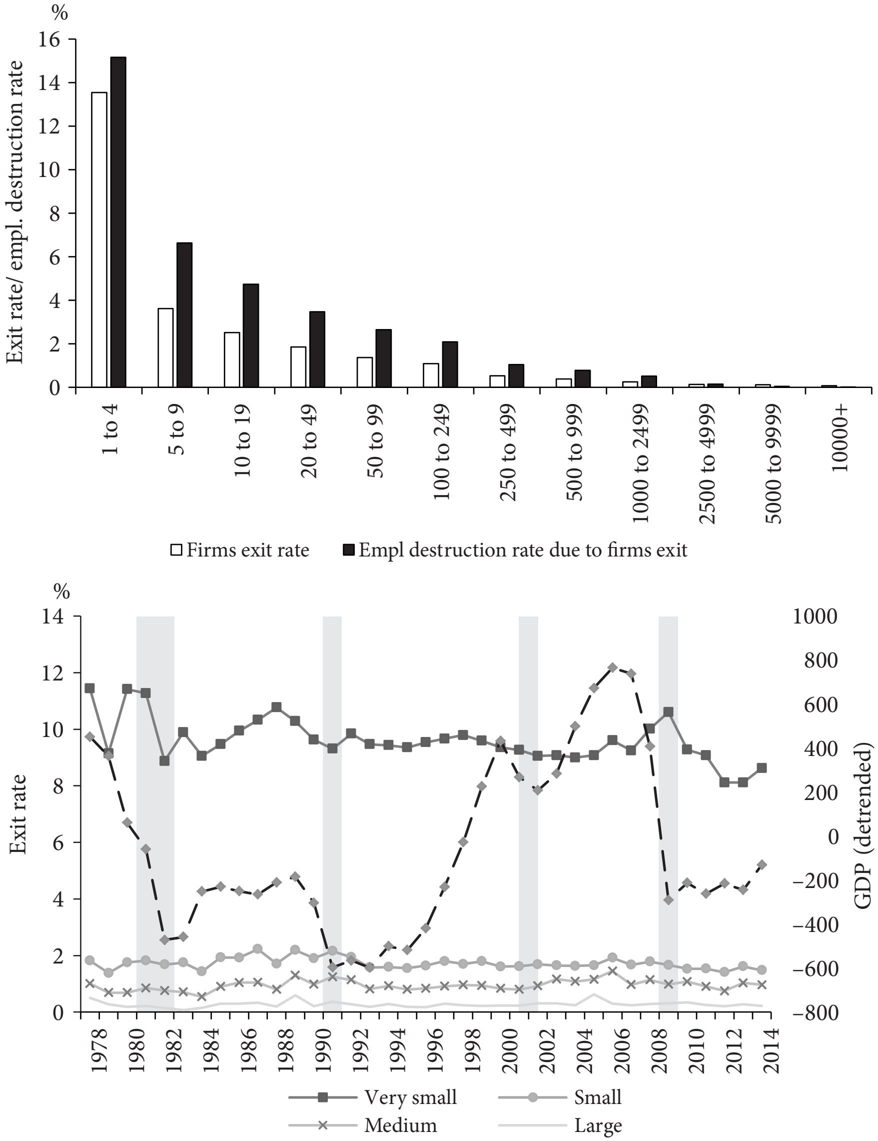
Note. Data from the Business Dynamics Statistics database

Figure 1. Distribution of companies according to their employment size, expressed in terms of the number of firms (left) and total employment (right)

panies (500 workers and more), while very small companies (which employ up to 19 workers) account for 20% of the total.

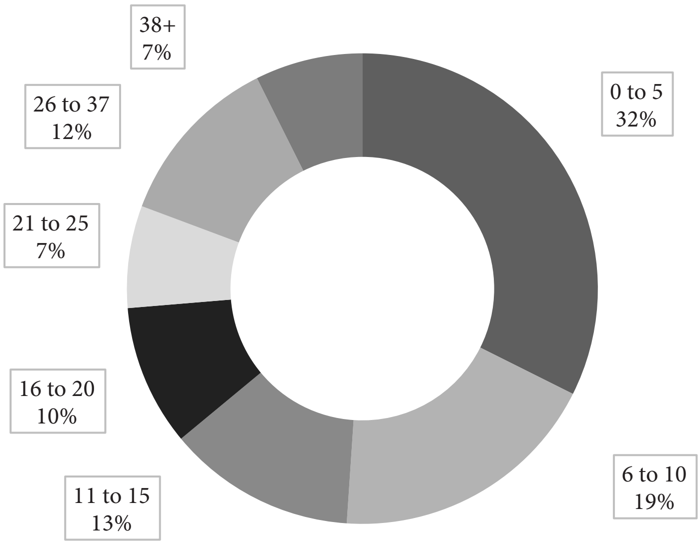
Figure 2 (left) shows the exit rate by firm size. The exit rate has been highly dependent on the size of the company. While for the smallest entities (which have employed between 1 and 4 workers) the average exit rate accounted for almost 14%, for the largest companies it was almost negligible. For each size category the average employment destruction rate has been higher than the exit rate itself, implying that the average employment in exiting firms has been higher than the average for all firm sizes. Moreover, for small, medium and large companies the exit rate has been relatively stable across time (around the level of 1.7%, 0.9%, and 0.3% respectively; see Figure 2, right). For very small entities (employing up to 19 workers) the exit rate has been diminishing across time (amounting to 11.4% in 1978 and 8.6% in 2014). The contemporaneous correlation between exit rate and output growth has been statistically insignificant, which is consistent with the findings of Campbell (1998). Moreover, in line with Campbell (1998), counter-cyclicality of exit rate can also be observed, however the details of the pattern are different from his work. For quarterly data, Campbell reported the correlation between exit rate and GDP growth four quarter hence to be 0.51, with the relation being persistent also eight quarters ahead. While the BDS data also supports the counter-cyclicality hypothesis, the positive correlation of 0.59 can be tracked for the GDP data lagging (and not leading – as reported by Campbell) the exit rate by 2 years. The potential reasons for this discrepancy involve data frequency (yearly data has been used for this study, while Campbell, 1998 used quarterly data) and the time scope of the analysis (1977–2014 for this study and 1972q2–1988q4 for the study of Campbell).

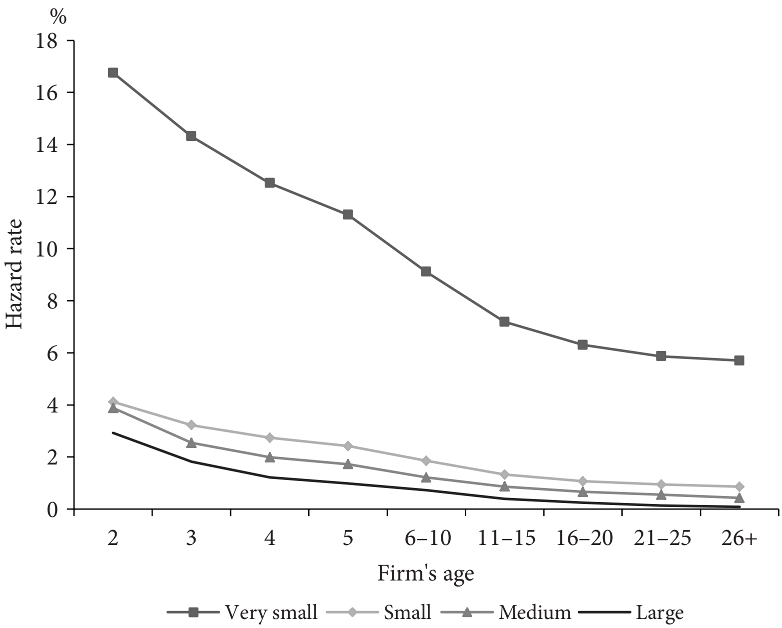
In terms of age, in 2014 the majority of firms (51%) were not 'older' than 10 years (see Figure 3, left). The probability of a firm exiting in a specific year since its entry, conditional on surviving up to this year (hazard rate) shows a strong dependence on the size of a company (see Figure 3, right). While for young companies of the smallest size (i.e. employing up to 19 employees) the risk rate has exceeded 15%, for larger firms it is in most cases smaller than 5%. In line with the intuition, the risk rate is declining over a firm's age. This might be due to the fact that weakest entities are being washed out in the first year(s) of their activity. Therefore the higher the age, the more resilient are the companies that have managed to survive up to that point.



Note. Data from the Business Dynamics Statistics database. Original data on GDP from the Bureau of Economic Analysis (GDP in billions of chained 2009 dollars), transformed by the author to remove a linear trend. Classification of firms: very small (1–19 empl.), small (20–99 empl.), medium (100–499 empl.), large (500+ empl.). On the right plot periods of economic contraction have been shaded

Figure 2. Exit rate and employment destruction rate according to firm size, average over 1978–2014 (upper figure) and exit rates across firm size and time (lower figure)





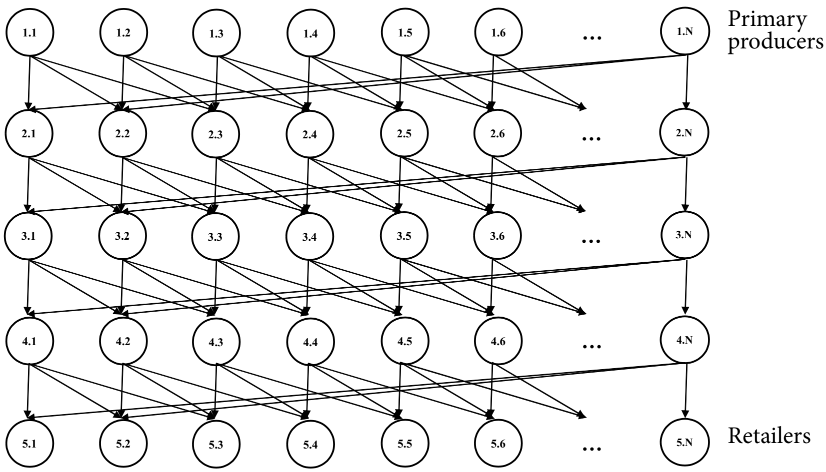
Note. Data from Business Dynamics Statistics database

Figure 3. Distribution of companies according to a firm's age in 2014 (in years, upper figure) and hazard rate (in years, lower figure)

The abovementioned patterns observed in the U.S. data have been used in this study to verify the results obtained from a production network-based simulation model.

2. Model description

For the purpose of the study, the agent based model in Battiston et al. (2007) has been replicated. In the model firms are connected with each other to form $M = 5$ layers of the production network (see Figure 4). Each company is connected to exactly three upstream firms (suppliers) and three downstream firms (customers), except firms at the first level (primary producers) who do not have any suppliers and firms at the final level (retailers) who sell their products directly to the market and therefore are connected only to upstream companies.



Note. In the simulation there are $N = 100$ companies at each level of the production network

Figure 4. Structure of the simulated network

The network of connections, once determined, stays unchanged for the whole simulation time. In each period (year) each firm makes decisions in the following steps: (1) deciding about the amount to produce and, based on this amount, sending orders to suppliers; (2) calculating the expected output based on feedback received from suppliers; (3) producing and delivering the manufactured production to customers; (4) receiving payments from customers and paying for intermediate goods. During each step companies get affected by two types of shocks which may lead to their bankruptcy (see the details below). After failure a firm stays inactive for 3 periods and returns to the market afterwards. During this inactivity time, a firm's suppliers and customers do not search for any other contractors, but allocate their orders and supplies to their remaining connections. Since no other entries take place, the size of the network is limited to the initial number of companies and shrinks temporarily due to bankruptcies taking place. More specifically, particular stages of the model are designed as follows.

In each period, each company i from level K decides on the amount to produce based on orders received from companies downstream and on its own production capabilities. For companies at the final level (retailers; the final level of the network is denoted as M) desired production, denoted $Y^{(d,M)}(t)$, is equal to linear production technology:

$$Y^{(d,M)}(t) = \theta A^{(M)}(t),$$

where $A^{(M)}(t)$ denotes assets of firms from level M expressed in the matrix notation and θ (technology) is equal to 1.

Orders are then equally allocated among suppliers (i.e. one third to each supplier). Given this, the desired production of upstream firms is the minimum of orders received and production capabilities:

$$Y_i^{(d,K)}(t) = \min\left\{\theta A_i^{(K)}(t), \sum_{j \in V_i^C} \frac{1}{3} Y_j^{(d,K+1)}(t)\right\},$$

where V_i^C denotes the set of customers of firm i.

After the top-down process of sending an order finishes, expected production is determined in the opposite direction, i.e. starting from the primary producers. If the orders sent by a company cannot be met due to the production limits of upstream firms, the expected output of a company would be smaller than the desired one and the expected supplies to its customers are scaled back proportionally.

After the expected output is set for all companies, the production process begins and proceeds upward, starting from the lowest level of the network. At this stage each company is faced with the probability q a production failure shock. If that happens, a company receives orders from its suppliers, but – due to a negative shock – is unable to deliver any production to its own customers. The shock then propagates down the network, since a scaled back amount of intermediate goods would force the companies downstream to limit their own production output.

When the production and delivery reach the final stage of the network, payments take place. Since the payment stage follows the delivery stage (instead of payments being settled immediately), a full trade credit is assumed to be provided by producers to their customers. The profit (or loss) of a company in each period is determined by the price of the offered goods, amount of actual delivery, denoted $Y_i^{(s,K)}$, and the costs of production:

$$\pi_i^{(K)}(t) = u_i(t) Y_i^{(s,K)}(t) - C_i^{(K)}(t).$$

For each company in each period the price u is a sales price shock of firm i in level K, which is independently and uniformly distributed within the interval $\left[1-\delta, 1+\delta\right]$. Costs, $C_i^{(K)}(t)$ can be classified to one of two parts.

The first one, $C_i^{(s,K)}(t)$ represents the cost of intermediate goods and is therefore proportional to the actual amount of inputs received:

$$C_i^{(s,K)}(t) = c_s \sum_{j \in V_i^S} Q_{ij}^{(K,K-1)} u_j(t) Y_j^{(K-1)},$$

where V_i^S denotes the set of suppliers of firm i, c_s denotes the ratio between the price levels at level $K-1$ and K (assumed to be the same for all K) and $Q_{ij}^{(K,K-1)}$ denotes a fraction of the production of firm j that this company supplies to firm i.

The second part represents the costs of production factors used to process the inputs, regardless of whether the expected output is actually produced:

$$C_i^{(r,K)}(t) = c_r \sum_{j \in V_i^S} Q_{ij}^{(K,K-1)} Y_j^{(e,K-1)},$$

where c_r denotes a constant proportion between the cost and expected output.

The realized profit or loss of a company is accrued to its assets, consequently affecting the production capabilities of the company in the following periods:

$$A^{(K)}(t+1) = \rho A^{(K)}(t) + \pi_i^{(K)}(t),$$

where $1-\rho$ accounts for the depreciation rate of assets.

If a company incurs a loss of more than β share of its assets, it becomes bankrupt and does not make any payments to its suppliers:

$$\frac{\pi_i^{(K)}(t)}{A^{(K)}(t)} < -\beta.$$

The firm then leaves the market. After τ periods a bankrupted company is replaced by a new firm with initial assets of A_{entry}. Each time a firm that exits is replaced by a new entry, thus the structure of connections stays unchanged.

For any further details concerning the description of the model, please see Battiston et al. (2007).

3. Model calibration

There are 11 parameters in the model (M, k, θ, q, δ, c_s, c_r, ρ, β, τ, A_{entry}). Three of them (q, ρ, β) were calibrated to match three statistical moments observed in the U.S. data. The probability of no output being delivered to customers (q) was used to calibrate the value of average aggregate output growth in the model. The higher the value of q, the higher the probability that a company will fail to produce the output it promised to deliver to its customers. The case of no-delivery causes a domino effect for downstream firms, as they lack inputs to meet their own production plan. As a result, the higher value of q negatively impacts the growth rate of the aggregate output. Since there is no technological shock in the model that would be responsible for causing a positive average dynamics of the model, the average aggregate growth rate is normalized to 0%. The bankruptcy threshold (β) was used to match the average exit rate observed in the U.S. data (2.79%). The higher the threshold, the easier it is for the company to go bankrupt, which directly implies a higher exit rate. Finally, a depreciation factor (ρ) was used to calibrate the 5-year unconditional survival probability (69.24%). The higher the depreciation rate, the slower the firm's assets accumulate along time, which negatively influences the chances for survival. The remaining parameters were set at the level proposed by Battiston et al. (2007); for details see Table 1.

Table 1. Parameterization of the model

Parameter	Description	Value	The way of determining the value
M	Number of levels of the supply chain	5	Battistion et al. (2007)
k	Number of suppliers of each firm in the model	3	Battistion et al. (2007)
$\boldsymbol{\theta}$	Production technology	1	Battistion et al. (2007)
$\boldsymbol{q}$	Production failure probability	0.02	Calibrated to match the normalized growth rate (0%)
$\boldsymbol{\delta}$	Price interval width	0.2	Battistion et al. (2007)
$\boldsymbol{c_s}$	Input cost factor	0.35	Battistion et al. (2007)
$\boldsymbol{c_r}$	Processing cost factor	0.35	Battistion et al. (2007)
$\boldsymbol{\rho}$	Depreciation factor (depr. rate = 1-rho)	0.97	Calibrated to match the average exit rate observed in the US data
$\boldsymbol{\beta}$	Bankruptcy threshold	0.2	Calibrated to match the average 5-year survival prob. observed in the US data
$\boldsymbol{\tau}$	Inactivity time	3	Battistion et al. (2007)
$\boldsymbol{A_{entry}}$	Entry level of net worth	10^{-1}*avg(A)	Battistion et al. (2007)

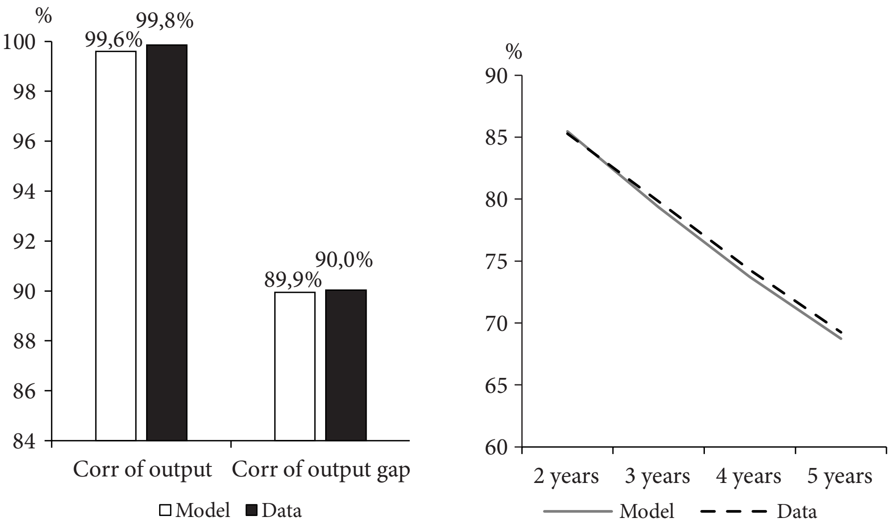
The calibration process is to minimize the sum of weighted absolute differences between the value of the respective moments as observed in the model and in the data. The inverse variances of the respective absolute differences have been used as weights.[6] Table *2* compares the targeted mo-

[6] Taking into account the limited capabilities of the hardware used for simulation, a two-step optimization procedure has been applied to calibrate the parameters. First, a minimum

ments between the data and the model. The calibrated parameters allow the model to well match the statistics from the data.

Table 2. Results of model calibration (in %)

Moment	Value in the model	Target value
Average aggregate growth	235	0
Average exit rate	2.81	2.79
5-year unconditional survival probability	68.74	69.24



Note. Output gap calculated as a detrended logarithm of output

Figure 5. Performance of the model: unconditional survival probability (left), autocorrelation of the output and output gap (right)

In order to externally verify the model calibration, Figure 5 compares several non-targeted statistics from the model and the data. The figure shows that the model successfully reproduces the persistence of both aggregate production and the output gap.[7] Moreover, even though only the

value on a wider three dimensional grid of parameters' combinations was found as an approximation of the optimum. Then a second run at a more precise grid was performed (centered around the parameters' values determined in the first step) that resulted in the fine-tuning of the initial result.

[7] Defined as detrended logarithm of output.

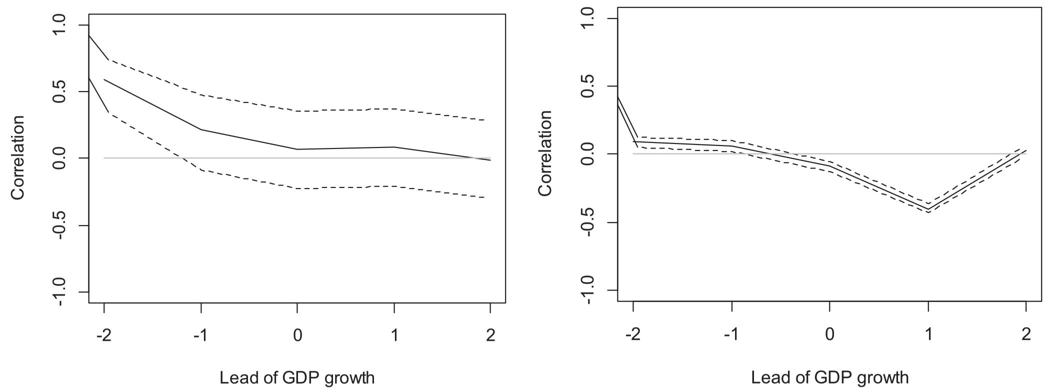
5-year survival rate has been targeted during the calibration process, the model successfully reproduces also the 2- to 4-year survival rates.

4. Results

In this section, the calibrated model has been tested for its ability to reproduce firms' exit patterns observed in the U.S. data, specifically a correlation between exit rate and output gap as well as a correlation between the growth rate of aggregate output and the average size of the exiting firm.

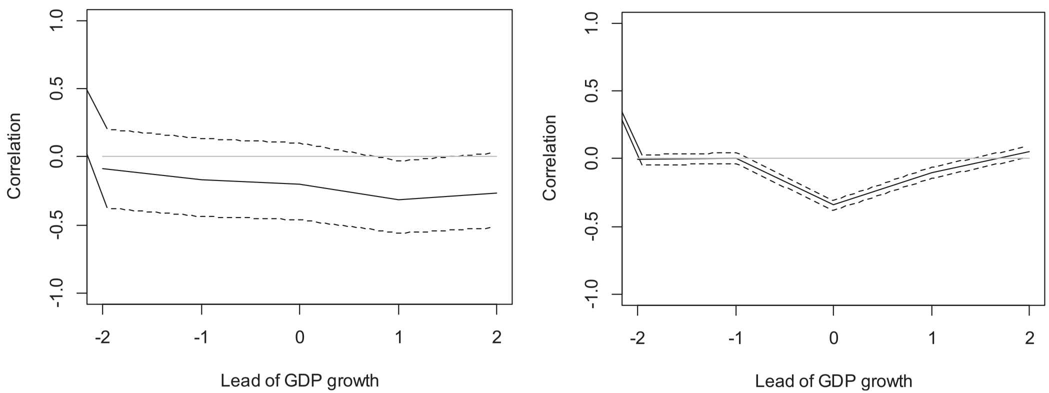
The model is partly able to reproduce the counter-cyclical pattern of exit rate (see Figure 6). The data show a countercyclical pattern with output growth lagged by 2 periods being positively correlated with the exit rate (correlation coefficient equal to 0.59). For the other data reported in Figure 6 there seems to be no significant correlation between the variables. In the model, the positive correlation between output growth lagged by 2 periods and the exit rate is also statistically significant; however the level of correlation is considerably smaller than in the data (0.09). Contrary to the data, the contemporaneous correlation as well as the correlation with output leading by one period is negative and statistically significant. This pattern in the model is driven by the following mechanism. Negative output growth in the model is driven by a production failure (and potential bankruptcy) hitting a relatively big company or companies. Even if the sizable assets of these companies allow them to survive the shock, by failing to supply their customers, the shock faced by these firms can cause bankruptcies in downstream companies (and therefore a higher exit rate).

Similarly to the exit rate, the model is also partially able to track the data in terms of the co-movement between the output growth and the average size of the exiting firm (see Figure 7). In the U.S. data the point estimate of the correlation between the two variables is negative but not statistically significant, except for the case in which GDP growth leads the average size of the exiting firm by one year. In this case the correlation coefficient is statistically significant and the point estimate is –0.32. The negative correlation in this case is matched by the model, although with a smaller magnitude (correlation coefficient equal to –0.1). Consistently with the data, the model suggests no significant relationship between the size of the exiting firm and lagged output. Since the endogenous business cycle in the model is driven by firm bankruptcy, this results in a negative contemporaneous correlation between the average size of the exiting firm and output growth.



Note. Confidence level in both graphs equal to 0.9

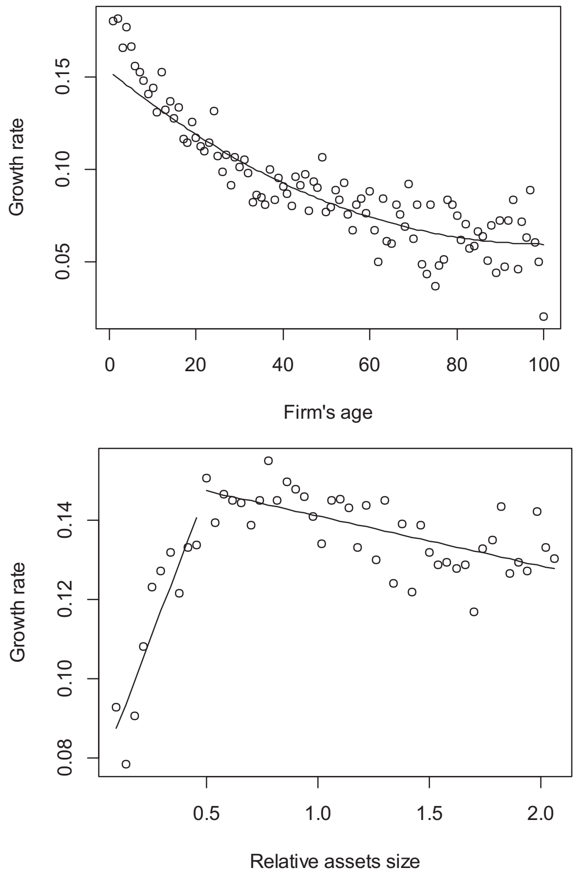
Figure 6. Correlation of GDP growth and exit rate in the data (left) and in the model (right)



Note. Confidence level in both graphs is equal to 0.9

Figure 7. Correlation of GDP growth and the average size of the exiting firm in the data (left) and in the model (right)

Additionally to firms' exit patterns, Figure 8 shows the ability of the model to match two other stylized facts documented in the literature, i.e. a negative correlation between the firm's growth rate and age, as well as between the firm's growth rate and its size; see Dunne, Roberts, & Samuelson (1988); Evans (1987); Hall (1987); Coad (2009); Gomis & Khatiwada (2017). In line with the literature, the firm's output growth in the model is decreasing with the firm's age. Since the failure to produce and deliver the



Note. The last observation in the left graph represents all entities of age 100 and more. The relative assets size in the right graph is calculated as a relation between the firm's assets and the average assets in a given period. The assets values for all companies in all periods are then assigned to 50 data bins according to relative size. The last bin groups companies of relative size 2.1 or higher

Figure 8. Correlation between the growth rate of output and the firm's age (upper figure) or relative assets' size (lower figure)

expected output affects not only the firm in question, but also its downstream customers, it is the downstream companies that are most vulnerable to negative assets shocks and – as a result – bankruptcy. Companies at the 5th level get hit by shocks that originate in every other layer, i.e. much more often than primary producers. On the other hand, the growth rate of output of each company depends on the quantity of orders placed by its customers. 'Newly born' downstream companies tend to place relatively small orders (due to their small relative size); what negatively affects growth prospects in mature upstream companies, whose assets (and therefore production capacities) would allow for much higher production than the orders they received.

For the relation between output growth rate and relative assets' size, the model suggests two different forms of relationship, depending on the level of the latter variable. For lower levels of the variable (up to around 0.5 of relative firm size) the correlation is positive and is mainly driven by differences in growth rate at different levels of the production network. Due to a high margin at each level of the network (equal to $1-c_s$), the nominal values of assets of downstream companies tend to be of a higher magnitude than it is the case for their upstream contractors. A positive relationship between production level and growth rate results in the correlation between growth rate and assets size being positive up to some point. This rationale prevents the model from replicating the above-mentioned negative correlation between assets' size and growth rate documented in the literature. On the other hand, when downstream companies are compared with each other, the direction of the correlation turns negative. The higher the assets, the lower is a company's potential to grow (since the growth rate depends on the size of orders, which depend on the size of firms' customers).

Conclusions

The results presented in this study show that the model of Battiston et al. (2007) is, to a certain extent, able to match the firms' exit patterns observed in the U.S. data. One the one hand, the model can track the counter-cyclicality of the exit rate, measured as a positive correlation between output growth lagged by 2 periods (years) and the exit rate. It is also partially able to track the co-movement between the output growth and the average size of the exiting firm; however the pattern suggested by the model is stronger than the one observed in the data. On the other hand the calibrated model reproduces a counterfactual strong negative contemporaneous correlation between the average size of the exiting firm and the output growth. Finally, the negative correlation between the growth rate of output and the firm's age that is documented by the literature is well matched by the model, while the negative contemporaneous correlation between relative assets' size and the output growth is only partially captured. The challenges the model faces in replicating the above-mentioned stylized facts seem to be the result of a series of simplifying assumptions used in the calibrated model. Rigid connections between the companies, simplified decision-making process at the level of a firm (concerning i.a. the size of the order and the algorithm of distributing the order among suppliers) and the too simplistic firm entry procedure seem to constitute the most evident limitations of the study and the room for further extensions. In terms of the structure of connections, one of the concepts that was not considered by Battiston et al. (2007), yet seems worth taking into account in further work deals with the issue of the embeddedness of firms, see Uzzi (1996) or Uzzi (1997). Moreover, in this study the assets of a company are used as a measure of a firm's size, while in the data – due to the limitation of the BDS data set – firm size is measured and based on the number of employees. A re-examination of the findings when the data on firms' assets are available has been left for further research.

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